Reputation For Two Audiences: Rating Agencies, Auditors, and Issuer-Pays Markets

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Abstract

This paper considers a dynamic model of reputation formation with two audiences. The motivation for studying this model comes from the issuer-pays feature of many certification intermediary markets such as auditors and credit rating agencies. In financial markets, certification intermediaries like auditors and credit rating agencies acquire information about the financial health of a firm. The information these intermediaries provide is used by investors in order to mitigate information risks. This paper investigates whether in a litigation free world reputation concerns can lead to a socially efficient outcome, that is, whether reputation concerns can provide incentives for auditors to expend effort in order to produce high quality auditing. This paper also investigates whether competition among the auditors improves reputational incentives.

Reputation of a certification intermediary is modeled as the market belief about his informativeness, which is exogenously given. There are two types of auditors, "informative" and "uninformative". In a monopoly set up, under imperfect monitoring, the informative auditor is diligent only for a low range of reputation. Gains from reputation shrink as the market becomes almost convinced about the auditor's type which leads to a continuum of threshold equilibria. The desired "high effort" equilibrium, which is also socially efficient, occurs only under the restrictive assumption of perfect monitoring. Comparing a duopoly and a monopoly model, I show that the range of reputation for which diligence can be sustained (when cost of diligence is small) is larger under monopoly. Reputation incentives are further weakened in a duopoly set up when firms have private information about the quality of their projects.

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1 Introduction

Certification intermediaries are information intermediaries who play an important role in mitigating information risk for a buyer by acquiring information about a seller. The most interesting feature of many of these institutions is the two audience feature resulting from the issuer-pays payment structure. In an issuer pays payment structure the subject of opinion is the client itself. Examples of such certification intermediaries are: laboratories in markets for industrial products (a famous example is Underwriters Laboratory), schools rating the ability of students; investment banks and underwriters evaluating the quality of firms that want to raise capital etc. Bond-rating agencies and auditors are the most significant ones in the context of financial markets.

This paper presents a model of reputation in the context of issuer-pays markets. The model is particularly interesting because of its relevance in the domain of certification intermediaries in financial markets. Both the rating agencies and the big audit firms came under severe public scrutiny through lawsuits, government investigations, and media attention after the Enron scandal in 2001 and the financial crisis of 2007-2008. The Credit rating firms are partly blamed in the major corporate failures of the last decade for their lack of diligence in identifying credit problems. The auditors on the other hand are often alleged to be involved in major accounting scandals, Enron(2001) being the most prominent example, leading to the demise of its auditor Arthur Anderson.

The issuer-pays payment structure has been vehemently criticized by the regulators because of its inherent conflict of interest. The proponents of issuer-pays model however argue that the conflict of interest can be mitigated through reputation concerns. The underlying idea is that if buyers determine that a certification is of low quality, they will stop crediting the certification intermediary, and its business will lose value. Adherents of the reputational capital model believe that market discipline, in the form of fear of loss of reputation, does or at least can provide the right incentives for high-quality certifications. They believe that poor performance is deterred by the prospect of loss of reputation, and accordingly do not believe that liability or other ex post legal remedies are an appropriate adjunct to the reputational mechanism. The standard positive theory of intermediary behavior yields the proposition that regulatory behavior in these markets are superfluous at best and distortionary at worst in terms of incentives to act diligently.

Another criticism of these markets is the high degree of market concentration. The rating agency market comprises of three big firms, Moody's, S&P, and Fitch while the audit market is dominated by the "Big Four": PWC, KPMG, Ernst and Young, and Deloitte. The prevalent argument against market concentration is that competition stimulates reputation building behavior as dissatisfied clients have the option of switching auditors. Better reputation is typically associated with higher market shares and higher profits and competition is typically perceived to be a devise to sustain high effort.

For concreteness I discuss the problem in an audit market setting. High quality auditing is a central component of corporate governance and what leads to high quality auditing is an open question. The two forms of incentives that has been discussed in the Accounting literature is litigation incentives and reputation incentives. The litigation incentives work in a straight forward way by making the auditor legally liable for the opinion he issues. Reputation incentives on the other hand takes a rather longer and indirect way of reaching the auditor and the question of interest is whether auditors' incentives to maintain reputation sufficiently powerful, absent litigation?

This paper seeks answers to the following questions. First, does even a well-functioning reputation mechanism provide enough incentives for the auditors to produce high quality auditing and lead to a socially efficient outcome? Second, does competition among the auditors indeed improve reputational incentives? In order to answer these questions I use a stylized model that solely focuses on the role of reputation as a devise to sustain high effort.

This model of reputation incorporates both unobservable types, i.e. whether the auditor is informative, and unobservable actions, that is, whether the informative auditor exerts due diligence to acquire more relevant information. In the model, risk neutral entrepreneurs or firms are endowed with projects and investors who are also risk neutral are endowed with cash. Investors face a new firm each period as an investment prospect. The outcome of the project can be "good" or "bad" and the investor has no information about the quality of the project. There is an auditor who can acquire information about the quality of the project and is hired by the firm for a report or a rating. The auditor if hired receives his payment upfront. The quality of information depends on how much effort the auditor expends. The auditor has short term incentives for not being diligent as effort is costly and his revenue comes from the firm. However in order to have value to the firm, his reports must also appear credible to the investors.

The auditor can be "informative" or "uninformative" and the auditor's type is private information. The uninformative auditor can not improve informativeness by being diligent. An informative auditor's evaluation on the other hand are inherently more informative than that of an uninformative auditor and he can choose to improve precision through diligence. Reputation of the auditor has been modeled in the usual way, where reputation is both the investor and the firm's posterior expectation that the auditor is "informative".

Firms and investors only observe noisy signals about how much information the auditor acquires, and beliefs about the auditor's reputation is revised using Bayes' rule. If the auditor's reputation is determined once and for all then the informative auditor has no incentive to exert diligence.

The main results of this paper are captured in a simple two period model. In a monopoly set up, under imperfect monitoring, the informative auditor is diligent only for a low range of reputation. The auditor has low incentives to expend effort when there is no immediate credible threat of termination and the point beyond which users prefer a non diligent "informative" auditor to no intermediary. There can still be reputation gains from diligence through increased expected future payoff. However, gains from reputation shrink as the market becomes almost convinced about the auditor's type which leads to a continuum of threshold equilibria. In a threshold equilibrium, the monopolist stops exerting effort beyond a threshold reputation. This result invalidates the claims of the reputational capital model, which predicts that the auditor will be diligent whenever he is hired. The desired "high effort" equilibrium, which is also socially efficient, occurs only under the restrictive assumption of perfect monitoring. This theory thus predicts that intermediary failures may occur even in the most successful certification markets.

In this paper I also analyze a duopoly model to investigate how competition affects reputation building behavior. The duopoly model has two identical auditors whose types are not known. Each period only one auditor is hired by the firm. The hired auditor's actions determine the probability that he is hired next period and the payoffs he receives if hired. Competition drives down the prices these auditors can charge in a Bertrand competition and hence reduces expected future payoff of the auditors. This in turn reduces their incentives to expend effort which leads to low quality of certification. The situation worsens when firms have private information about the quality of the project they own.

Preview: The rest of the paper is organized as follows. Section 2 discusses related research. Section 3 presents the dynamic model along with strategies, beliefs and equilibrium concept. Section

4 presents the results of the two period monopoly model and Section 5 analyses the two period duopoly model. Section 6 provides brief discussions on report contingent payments, investor pays vs. issuer pays model and also contains a discussion on the infinite horizon version of the model. Section 7 concludes.

2 Related Literature

This paper contributes to three strands of literature: the literature concerning auditors' behavior, the literature concerning rating agency behavior and the reputation literature. The key modeling features of this paper are the two audience model, the reputation dynamics and competition among auditors.

My paper shares some modeling similarities with Narayanan (1994), Dye et al. (1990) and Dye (1993). Dye et al. (1990) look at a model of audit market and examine how auditors and their clients respond to report contingent audit contracts. Narayanan (1994) compares audit quality under "joint and several" liability rules and "proportionate liability" rules and suggests that switching from joint and several liability to proportionate liability may actually increase audit quality. Dye (1993) also builds a model of audit market relating auditor liability and auditing standards. He shows how auditing fees depend on the informational value of the audit and the option value of the claim financial statement's users have on auditors wealth in the event the audit is determined to have been substandard.

Reputation dynamics for auditors has always been a question of interest to the Accounting researchers. In a recent paper Douglas and Skinner(2012) investigate the importance of auditors' reputation for delivering quality using data from the litigation free country Japan, while my paper theoretically explores the reputation dynamics in a litigation free world. The reputation models in the auditors' behavior literature are fairly new and largely unexplored. Corona, Randhawa (2010) studies a two period game of repeated interactions between a manager and an auditor. They investigate reasons and circumstances under which reputational concerns induce an audit firm to misreport. With long lived managers their paper they also incorporate the flavor of relationship building in their paper. Unlike existing papers in the auditors' behavior literature my paper also addresses the issue of competition in audit market with a dynamic model of reputation formation.

The paper incorporates the following institutional features discussed in Sunder (2003). First, the paper builds on the assumption that outside independent auditors help inform investors about the integrity of financial statements. Second, the results of an audit service is both imperfectly correlated with audit quality and rarely observed by the managers and investors. The imperfect correlation aspect has been captured by a model of two sided imperfect monitoring in my paper. However my model does not explicitly assume that the outcome of an audit service is rarely observed. This feature can be easily incorporated by assuming that the probability with which a true outcome is revealed at the end of any period is strictly less than one. In this event the results will still go through and the additional assumption will only have a scale effect. In addition this exercise will give us insights on how probability of the truth being discovered affect reputation incentives. The results will provide a formal representation of Sunder's (2003) argument against competition as a reputation building device. Third, the model also captures the switch to an analytical review intensive production function by introducing a choice of effort for an auditor who has private information. This paper builds on the features of private information and uncertainty (Akerlof, "Market for Lemons") by adding moral hazard to the model to formally analyze the reputation dynamics.

Papers that deal with rating agencies and need to be mentioned in reference to this paper are Skreta and Veldkamp (2009) and Sangiorgi and Spatt (2012). Skreta and Veldkamp show that increase in asset complexity leads to increase in credit shopping. Competition worsens the problem in their framework too. Sangiorgi and Spatt develop a rational expectations model where a debt issuer purchases credit ratings(s) to provide useful information to investors and attract demand. The issuer purchases rating(s) sequentially and decides which to disclose. Their analysis emphasizes the importance of opacity about the contracts between the issuer and rating agencies, leading to potential asymmetric information by investors about which ratings have been obtained. My paper in contrast, rules out certification shopping and focuses on isolating the effect of competition with reputation being the sole motivator.

Two papers that are closest to my paper in terms of modeling similarities are by Mailath and Samuelson (2001) and Mathis, McAndrews and Rochet (2008). Mailath and Samuelson (2001) builds a model of reputation in the context of experience goods market which is a one-audience market. They examine a market in which firms face a moral hazard problem: they have a short-term incentive to exert low effort, but could earn higher profits if it were possible to commit to high effort. They view reputation as a commitment device that allows firms to solve the moral hazard problem. There are two types of firms. The "competent" firms has a choice between exerting high effort and low effort while the "inept" firm can only exert low effort. There are occasional exits and firms that replace the existing firm may be competent or inept. These replacements are crucial in sustaining a high effort equilibrium. My model assumes away the occasional exits and shows that a high effort equilibrium exists only under restrictive assumptions. This result is in similar spirit to the result obtained by Mailath and Samuelson in the context of one-audience markets. I also assume that the "informative" auditor is inherently better than the "uninformative" auditor even if he does not exert diligence. This feature of the model helps to avoid the no effort equilibrium observed in Mailath and Samuelson's paper. In their paper, they assume that the "competent" firm produces the same quality of output as the "inept" firm produces, if he does not expend effort. This assumption is crucial for the "no effort" equilibrium they observe. Moreover they do not discuss issues related to competition in their paper.

Mathis, McAndrews and Rochet (2008) deal with reputation building behavior by a Credit Rating Agency in a dynamic set up. They build on two important papers by Benabou and Laroque (1992) and Diamond (1989). While the paper by Benabou and Laroque examines the behavior of a Guru who can influence the behavior of uninformed investors, the paper by Diamond studies why new firms with insufficient reputation have to borrow from banks before they can issue direct debt. In the paper by Mathis, McAndrews and Rochet each period a new cashless firm wants to issue security for financing a complex investment project. The project quality is *a priori* unknown, including to the issuer himself. A Credit Rating Agency can observe the quality of the project and communicate it to the market through a rating. They show in a monopoly set up that reputation concern fails to prevent rating agencies from inflating ratings when rating complex products becomes a major source of income for the rating agency. My paper in some sense investigates a more fundamental issue by looking at the role of reputational incentives to find out the truth by exerting effort while Mathis, McAndrews and Rochet primarily investigate incentives to reveal the truth. By assuming upfront payment method instead of report-contingent payment I show that upfront payment may improve reputational incentives but does not provide complete cure to the problem. Moreover my analysis includes how competition among certification intermediaries affects reputational incentives to acquire information. I also formally model the case where firms have private information about the quality of their projects.

The vast literature on reputation that this paper relates to mostly deals with one-audience models. Holmstrom (1999) analyses a managerial incentive problem where the manager works to induce erroneous market beliefs that its ability type is higher than in reality. Initially the manager's talent is incompletely known to both the manager and the market. There are no explicit output contingent contracts but wages depend on expected output which essentially depends on the market's assessment of the manager's talent. The manager takes unobservable actions which gives rise to an incentive problem. In Board and Meyer-ter-Vehn (2009), reputation is modeled directly as the market belief about quality which is persistent and depends stochastically on the firm's past investments. They build their model in a continuous time framework. However quality improvements are made at discrete points of time. Quality at a point in time is determined by the firm's investment at the most recent technology shock, between shocks quality is constant. My model follows the existing literature in terms of how reputation is modeled. But the model differs from existing models of reputation through the two audience feature of the issuer pays markets. The two audience feature generates a self absorbing state at the lower range of reputation and can potentially lead a negatively sloped return function. Reputation incentives depend on the probability of reaching the self absorbing along with how much the firm values an informative auditor.

Horner (2002) shows how competition generates reputation building behavior in the context of "experience goods" markets. While Horner talks about a perfectly competitive market and emphasizes on the importance of outside options generated through competition, my model compares a duopoly model with a monopoly setting in an environment of price competition. My paper focuses on how competition shrinks expected future payoff and reduces reputation incentives for certification intermediaries.

While the reputation models largely focus on one audience markets, the two audience market models in the economic literature are predominantly static models. Bhattacharya and Ritter (1983) and Gertner, Gibbons and Scharfstein (1988) have this feature of two audiences in their models, however in a different context. They analyze situations where an asymmetrically informed auditor is motivated to communicate its privately known attribute but can do so only through channels or signals which convey directly useful information to competing auditors. This revelation to the competition serves to reduce the value of the private information held by the first auditor. Bhattacharya and Ritter (1983) develop their model in the context of a set of firms engaged in research and development rivalry. Pursuing R&D activity requires firms to raise external financing in the capital market. The model is structured so that the only way the informed firm can communicate its prospects to the capital market is through the disclosure of technological information of direct usefulness to competitors. The informed firm therefore faces a tradeoff between (i) reducing the value of its informational advantage, and (ii) raising financing at better terms. In Gertner, Gibbons and Scharfstein (1988) when a firm reveals information to the capital market, it often does so by a publicly observable action (such as a dividend) that reveals information to otherwise uninformed auditors in other markets (such as product-market rivals). These auditors then condition their behavior on this information, and this affects the profit (gross of financing costs) of the informed firm. Both these models are essentially two audience signaling models built in a static framework. This paper in contrast talks about an auditor who manipulates reputation for two audiences in a dynamic framework.

3 Environment

Players, actions and payoffs: This is a discrete time two period model where time is indexed $t = 1, 2$. There are three types of players; firms, investors and auditors. Firms and investors are short-lived while auditors are long-lived. Each period a cashless firm wants to raise capital for financing a project it owns. The investor, who faces the firm as an investment prospect, is endowed with cash of amount w. The auditor is equipped with an information acquiring technology which can acquire information about the quality of the project the firm owns.

Production takes place only if the entire w is invested in the project. The outcome of the project can be good(G) or bad(B) and the project quality is *a priori* unknown to the investor and the firm itself. A good outcome occurs with probability p and returns to the investor r_1 is strictly positive. With probability $(1 - p)$ the outcome is bad in which case the project yields a negative return $r_2 \in [-1, 0)$, to the investor. r_1 and r_2 is exogenously given.

Firms' gains as a function of investment is given by $f(.)$ where,

$$f(x) = \begin{cases} 1 & , x = w \\ 0 & , otherwise \end{cases}$$

Investors are risk neutral and they are faced with the following problem

$$max_{a\epsilon[0,1]} w[\pi(1 + ar_1) + (1 - \pi)(1 + ar_2)]$$

where π is the probability that the outcome is good and the maximization gives

$$a^* = \begin{cases} 1 & , \pi r_1 + (1 - \pi)r_2 \geq 0 \\ 0 & , \pi r_1 + (1 - \pi)r_2 < 0 \end{cases}$$

Define, $\overline{p}$ such that $\overline{p}r_1 + (1 - \overline{p})r_2 = 0$. $\overline{p}$ is the posterior probability (that the outcome of the project is good) that makes the investor indifferent between investing all her wealth and not investing. Clearly, if $p \geq \overline{p}$ the firm is capable of getting investment in the auditor's absence. However, for priors lower than $\overline{p}$ firms fail to obtain investment without certification. This paper focuses on the case where $p < \overline{p}$.

Informativeness of the auditor: The auditor's information acquisition technology generates publicly observable signals $s\epsilon\{g, b\}$, where g signifies a good outcome and b, a bad one. Auditors are of two types "informative" and "uninformative". The "uninformative" auditor's signals carry no additional information as he receives signals $s \in \{g, b\}$ independent of the firm's true type. The "uninformative" auditor is also the non-strategic type who generates g with probability p and b with probability $1 - p$.

The "informative" auditor's informativeness is captured by the parameters α and ϵ, where α is the probability that a bad project is given the signal g (false positive) and ϵ is the probability that a good project is given b (false negative). Assume that $p\epsilon < (1 - p)\alpha$, i.e. ex-ante the informative auditor is more likely to commit an error when the project quality is bad. [1]

Notice that α and ϵ are the parameters that depict the extent to which information is distorted. Higher are the parameters lower is the informativeness of the auditor. Consequently, the higher the parameters the lower is the credibility of the signal s. Suppose that the informative auditor is informative enough so that $Pr(G|g) > \overline{p}$ which in turn implies $Pr(G|b) < \overline{p}$.

The informative auditor can improve precision by being diligent for which he has to bear a small cost $c > 0$. By being diligent the auditor passes a bad project as good with a lower probability $\alpha' \in (0, \alpha)$ and passes a good project as good with probability 1[2]. Notice that the informative auditor's signals are useful to the investor even if he is not diligent. In case of perfect monitoring $\alpha' = 0$, that is, the informative auditor can perfectly distinguish between a good project and a bad project.

At any period t, reputation of an auditor, denoted θ_t gives the probability that the auditor is "informative".

Suppose $c > 0$ is small enough so that the cost of diligence is lower than the information gain, that is, the informative auditor exerting diligence is efficient for the society.

Timeline: The sequence of events is as follows. At the beginning of period 1 the auditor decides whether to be diligent by paying the cost c and acquires information about the project quality of the firm. He receives signal $s \in \{g, b\}$ depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest. If the investor invests her entire wealth, then the true project quality is revealed at the end of period 1. The investor does not observe project quality if no investment takes place. In period 2 the firm and the investor updates their belief about the auditor's type using Bayes' rule. The auditor posts a price P and the firm decides whether to hire the auditor[3]. If the auditor is hired the payment is made upfront. If the auditor is hired he decides whether to be diligent by paying cost c and acquires information about

[1] This assumption ensures that the return function for the auditor is increasing in reputation and the auditor has incentives to improve reputation to begin with. However this assumption is not crucial as far as the results of the paper are concerned. Whether the return function is increasing or decreasing in reputation also depends on the probability with which the uninformative auditor generates the signal g. The above assumptions capture conditions conducive to reputation building behavior and they are in place only for the ease of exposition.

[2] This is a simplifying assumption. The results do not change if the informative auditor commits an error even if he is diligent.

[3] The focus of this paper is to analyze the impact of reputation incentives and reputation should be the sole motivator in my model. Therefore, to set aside the issue of price signaling I assume that the auditor does not post a fee in the first period.

the project quality. He receives signal $s \in \{g, b\}$ depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest.

Strategies and beliefs: A stationary strategy for the auditor is a pair (P, γ), i.e. , a fee and a choice of effort. Formally, $\gamma : [0, 1] \to [0, 1]$, where $\gamma(\theta)$ is the probability that the auditor is diligent in period 1 and $P : [0, 1] \to \mathbb{R}$ gives the fee the auditor posts in period 2.

A firm's strategy is a hiring function h from $\mathbb{R} \times [0, 1] \to \{0, 1\}$, where $h = 0$ implies the firm does not hire the auditor and $h = 1$ implies the auditor is hired.

The investor's investment strategy $a^* : [0, 1] \times \{g, b, \phi\} \to [0, 1]$ depicts how much the investor invests from a wealth w as a function of reputation and observed signal s.

The belief function $\pi : [0, 1] \times \{g, b, \phi\} \to [0, 1]$ gives the probability that the project is good given signal s and $\pi(\theta, s)$ is calculated using Bayes' rule.

At the beginning of each period the investor and the firm observes a signal $s \in \{g, b, \phi\}$ and at the end of that period one of the following three outcomes is observed. A good(G) outcome is observed if a good project is financed, a bad(B) outcome is observed if a bad project is financed and no(N) outcome is observed if no investment takes place. We denote by $\varphi(\theta|s, i)$ or φ_s^i the posterior probability that the auditor is "informative" given signal $s \in \{g, b\}$, outcome $i \in \{G, B, N\}$ and prior probability θ. If the "informative" auditor is diligent with probability $\gamma(\theta)$, then the posterior beliefs are

$$\varphi_g^G(\theta) = \frac{\theta[\gamma(\theta) + (1 - \gamma(\theta))(1 - \epsilon)]}{\theta[\gamma(\theta) + (1 - \gamma(\theta))(1 - \epsilon)] + (1 - \theta)p} \tag{1}$$

$$\varphi_g^B(\theta) = \frac{\theta[\gamma(\theta)\alpha' + (1 - \gamma(\theta))\alpha]}{\theta[\gamma(\theta)\alpha' + (1 - \gamma(\theta))\alpha] + (1 - \theta)p} \tag{2}$$

$$\varphi_b^N(\theta) = \frac{\theta[p(1 - \gamma(\theta))\epsilon + (1 - p)\{\gamma(\theta)(1 - \alpha') + (1 - \gamma(\theta))(1 - \alpha)\}]}{\theta[p(1 - \gamma(\theta)\epsilon + (1 - p)\{\gamma(\theta)(1 - \alpha') + (1 - \gamma(\theta))(1 - \alpha)\}] + (1 - \theta)(1 - p)} \tag{3}$$

The maximum willingness to pay by the firm for an auditor's certification is:

$$v(\theta) = \theta[p\{\hat{\gamma}(\theta) + (1 - \hat{\gamma}(\theta))(1 - \epsilon)\} + (1 - p)\{\hat{\gamma}(\theta)\alpha' + (1 - \hat{\gamma}(\theta))\alpha\}] + (1 - \theta)p$$

where, θ is the auditor's reputation and $\hat{\gamma}(\theta)$ is the belief about the auditor's choice of effort.

The equilibrium concept is Perfect Bayes. The equilibrium consists of a hiring strategy by the firm, a choice of effort and a fee by the auditor, an investment strategy by the investor, a posterior function and an updating rule such that, players maximize their payoffs and beliefs coincide with actions.

Definition: Equilibrium consists of a hiring strategy h by the firm, a choice of effort γ and a fee P by the auditor, an investment strategy a^* by the investor, a posterior function π and an updating rule φ such that,

1. h is optimal for the firm.
2. γ maximizes expected lifetime payoff for the auditor.
3. P maximizes period 2 payoff.
4. a^* is optimal for the investor.
5. π is obtained using Bayes' rule.
6. φ satisfies (1)-(3).

4 Analysis

This section analyzes the monopoly model and characterizes equilibria under the assumption of imperfect monitoring and perfect monitoring.

Notice that the investor invests only if she observes g. The investor's decision depends on reputation(θ) in the following way. There exists a $0 < \underline{\theta} < 1$, such that the investor is indifferent between investing and not investing if the auditor is diligent and the observed signal is g. That is, $\pi(\underline{\theta}|\gamma = 1) = \bar{p}$. Thus for all $\theta \in [0, \underline{\theta})$ the auditor is not hired even if he puts effort. This is a self absorbing state and the auditor can never make it to the market once he falls into this region. Thus, the auditor has no incentive to be diligent if his reputation falls in the region $\theta \in [0, \underline{\theta})$. There exists another important threshold reputation given by $\overline{\theta} \in (\underline{\theta}, 1)$, such that, if the observed signal is g, the investor is indifferent between investing and not investing even if the auditor does not exert diligence. That is, $\pi(\overline{\theta}|\gamma = 0) = \bar{p}$. Therefore, in equilibrium the auditor must exert diligence for the range $\theta \in [\underline{\theta},\overline{\theta})$ in order to get hired. Now, whether the auditor puts effort beyond $\overline{\theta}$, is the question of interest. The desired "high effort" equilibrium that supports the reputation concern arguments is one where the auditor exerts diligence whenever he is hired by the firm.

The following three Lemmas characterize how reputation is revised following signals and outcomes.

Lemma 1: *$\varphi_g^G(\theta) > \theta$ and $\varphi_b^N(\theta), \varphi_g^B(\theta) < \theta$.*

Proof:

$$\varphi_g^G(\theta) - \theta = \frac{\theta(1-\theta)[\gamma(\theta) + (1-\gamma(\theta)(1-\epsilon) - p]}{\theta[\gamma\theta + (1-\gamma(\theta)(1-\epsilon)] + (1-\theta)p} > 0$$

$$\varphi_g^B(\theta) - \theta = \frac{\theta(1-\theta)[\gamma(\theta)\alpha' + (1-\gamma(\theta)\alpha - p]}{\theta[\gamma\theta + (1-\gamma(\theta)(1-\epsilon)] + (1-\theta)p} < 0$$

$$\varphi_b^N(\theta) - \theta = \frac{\theta(1-\theta)[p(1-\gamma(\theta))\epsilon + (1-p)\{\gamma(\theta)(1-\alpha') + (1-\gamma(\theta))(1-\alpha)\} - (1-p)]}{\theta[p(1-\gamma(\theta)\epsilon + (1-p)\{\gamma(\theta)(1-\alpha') + (1-\gamma(\theta))(1-\alpha)\}] + (1-\theta)(1-p)} < 0 \blacksquare$$

Lemma 1 shows that reputation is revised upwards when a good signal is followed by a good outcome. On the other hand a bad signal and a good signal followed by a bad outcome leads to downwards revision of reputation.

Lemma 2: *$\varphi_g^B(\theta) - \varphi_b^N(\theta) < 0$ when $\gamma = 1$ and $\gamma = 0$.*

Proof:

With $\gamma = 1$,

$\varphi_g^B(\theta) = \frac{\theta\alpha'}{\theta\alpha' + (1-\theta)p}$ and $\varphi_b^N(\theta) = \frac{\theta(1-\alpha')}{\theta(1-\alpha') + (1-\theta)}$.

$$\varphi_g^B(\theta) - \varphi_b^N(\theta) = \frac{\theta(1-\theta)[\alpha'(1-p) - p]}{[\theta\alpha' + (1-\theta)p][\theta(1-\alpha') + (1-\theta)]} < 0$$

$$\Rightarrow \varphi_g^B(\theta) < \varphi_b^N(\theta)$$

With $\gamma = 0$,

$\varphi_g^B(\theta) = \frac{\theta\alpha}{\theta\alpha+(1-\theta)p}$ and $\varphi_b^N(\theta) = \frac{\theta[p\epsilon+(1-p)(1-\alpha)]}{\theta[p\epsilon+(1-p)(1-\alpha)]+(1-\theta)(1-p)}$.

$$\varphi_g^B(\theta) - \varphi_b^N(\theta) = \frac{\theta(1-\theta)[\alpha - \alpha p^2 - \epsilon p^2 - p + p^2]}{[\theta\{p\epsilon + (1-p)(1-\alpha)\} + (1-\theta)(1-p)][\theta\alpha + (1-\theta)p]} < 0$$

$$\Rightarrow \varphi_g^B(\theta) < \varphi_b^N(\theta) \qquad \blacksquare$$

Lemma 2 shows that reputation falls more when a good signal is followed by a bad outcome in comparison to the fall in reputation followed by a bad signal.

Lemma 3: Given θ,

$$p\epsilon(\varphi_g^G(\theta)-\varphi_b^N(\theta))+(1-p)(\alpha-\alpha')(\varphi_b^N(\theta)-\varphi_g^B(\theta))\Big|_{\hat{\gamma}=1} > p\epsilon(\varphi_g^G(\theta)-\varphi_b^N(\theta))+(1-p)(\alpha-\alpha')(\varphi_b^N(\theta)-\varphi_g^B(\theta))\Big|_{\hat{\gamma}=0}$$

Proof:

Notice that $\varphi_g^G(\theta)$ is increasing in γ and $\varphi_g^B(\theta)$ is decreasing in γ, that is revisions are milder when the believed choice of effort is zero. For notational convenience let us denote the believed choice of effort as γ (instead of $\hat{\gamma}$) for the rest of the proof.

Let, $\varphi_b^N(\theta)\Big|_{\gamma=0} - \varphi_b^N(\theta)\Big|_{\gamma=1} = \triangle$.

Now if $p\epsilon > (1-p)(\alpha-\alpha')$, $\triangle > 0$.

Also, the difference between $\varphi_b^N(\theta)\Big|_{\gamma=0} - \varphi_g^B(\theta)\Big|_{\gamma=0}$ and $\varphi_b^N(\theta)\Big|_{\gamma=1} - \varphi_g^B(\theta)\Big|_{\gamma=0}$ equals $\triangle$ which is same as the difference between $\varphi_g^G(\theta)\Big|_{\gamma=0} - \varphi_b^N(\theta)\Big|_{\gamma=1}$ and $\varphi_g^G(\theta)\Big|_{\gamma=0} - \varphi_b^N(\theta)\Big|_{\gamma=0}$.

Since, $p\epsilon > (1-p)(\alpha-\alpha')$

$$p\epsilon(\varphi_g^G(\theta)-\varphi_b^N(\theta))+(1-p)(\alpha-\alpha')(\varphi_b^N(\theta)-\varphi_g^B(\theta))\Big|_{\gamma=1} > p\epsilon(\varphi_g^G(\theta)-\varphi_b^N(\theta))+(1-p)(\alpha-\alpha')(\varphi_b^N(\theta)-\varphi_g^B(\theta)\Big|_{\gamma=0}$$

Similarly, when $p\epsilon < (1-p)(\alpha-\alpha')$, $\triangle < 0$.

Also, the difference between $\varphi_b^N(\theta)\Big|_{\gamma=0} - \varphi_g^B(\theta)\Big|_{\gamma=0}$ and $\varphi_b^N(\theta)\Big|_{\gamma=1} - \varphi_g^B(\theta)\Big|_{\gamma=0}$ equals $\triangle$ which is same as the difference between $\varphi_g^G(\theta)\Big|_{\gamma=0} - \varphi_b^N(\theta)\Big|_{\gamma=1}$ and $\varphi_g^G(\theta)\Big|_{\gamma=0} - \varphi_b^N(\theta)\Big|_{\gamma=0}$.

Since, $p\epsilon < (1-p)(\alpha-\alpha')$

$$p\epsilon(\varphi_g^G(\theta)-\varphi_b^N(\theta))+(1-p)(\alpha-\alpha')(\varphi_b^N(\theta)-\varphi_g^B(\theta))\Big|_{\gamma=1} > p\epsilon(\varphi_g^G(\theta)-\varphi_b^N(\theta))+(1-p)(\alpha-\alpha')(\varphi_b^N(\theta)-\varphi_g^B(\theta)\Big|_{\gamma=0} \blacksquare$$

Suppose the auditor's reputation is such that the auditor is hired in the second period with probability 1. Lemma 3 implies that at any such reputation θ, the auditors expected payoff from exerting effort with $\hat{\gamma} = 1$, is strictly greater than his expected payoff from putting effort with $\hat{\gamma} = 0$.

Proposition 1: *There exists $\bar{c} > 0$ such that for all $c \leq \bar{c}$ there exist $\theta^1 \in [0, \overline{\theta}]$ and $\theta^2 \in [\overline{\theta}, 1)$ so that the following pure strategy profile constitutes an equilibrium. At t=2, the auditor is never diligent. The auditor posts the fee $P(\theta) = \theta[p(1-\epsilon) + (1-p)\alpha] + (1-\theta)p$ and is hired only if $\theta \geq \overline{\theta}$. The investor invests only if $\theta \geq \overline{\theta}$ and $s = g$. At t=1, the auditor is not diligent, i.e. $\gamma(\theta) = 0$ for $\theta \in [0, \theta^1)$. $\gamma(\theta) = 1$ for $\theta \in [\theta^1, \theta^2)$ and $\gamma(\theta) = 0$ for $\theta \in [\theta^2, 1]$. The investor invests only if $\theta \geq \theta^1$ and $s = g$.*

Proof:

At $t = 2$, if the auditor is hired the firm pays him first and then the auditor decides whether to be diligent. His decision in that period does not affect his payoff in period 2. Thus, not putting effort in period 2 is a dominant strategy for the auditor. Now $P(\theta) = \theta[p(1-\epsilon) + (1-p)\alpha] + (1-\theta)p = v(\theta)$ at $t = 2$, that is the auditor extracts all the rents as he gets to post the price.

Now, at $t = 1$, suppose the auditor exerts due diligence. With probability p the informative auditor meets a good firm and moves to a higher level of reputation, say to $\theta_G = \varphi_g^G(\theta)$. With probability $(1-p)$ he meets a bad firm and receives signal $s = b$ with probability $(1-\alpha')$. No investment takes place and reputation is revised to $\theta_N = \varphi_b^N(\theta)$. With probability α' the auditor makes a mistake and receives $s = g$. The investor invests and the true quality of the project is revealed. This pushes reputation down to $\theta_B = \varphi_g^B(\theta)$.

Therefore, the expected payoff by exerting effort in period 1 is given by, $pv(\theta_G)+(1-p)[\alpha' v(\theta_B)+ (1-\alpha')v(\theta_N)] - c$.

Now if the auditor does not put effort in period 1, he moves to a higher level of reputation θ_G with probability $p(1-\epsilon)$. With probability ϵ he makes a mistake and receives $s = b$ which is followed by no investment and reputation falls to θ_N. With probability $(1-p)$ he meets a bad firm and receives signal $s = b$ with probability $(1-\alpha)$. No investment takes place and reputation is revised to θ_N. With probability α the auditor makes a mistake and receives $s = g$. the investor invests and the true quality of the project is revealed. This pushes reputation down to θ_B.

Therefore, the expected payoff by not exerting effort in period 1 is given by, $p[(1-\epsilon)v(\theta_G) + \epsilon v(\theta_N)] + (1-p)[\alpha v(\theta_B) + (1-\alpha)v(\theta_N)]$.

The auditor will put effort in period 1 if,

$$c \leq p\epsilon[v(\theta_G) - v(\theta_N)] + (1-p)(\alpha - \alpha')[v(\theta_N) - v(\theta_B)] \tag{4}$$

The right hand side equals

$$[(1-p)\alpha - p\epsilon][p\epsilon(\theta_G - \theta_N) + (1-p)(\alpha - \alpha')(\theta_N - \theta_B)] \;\; if \; \theta_B \geq \overline{\theta}$$

Now from Lemma 1, $\varphi_g^G(\theta) > \theta$ and $\varphi_b^N(\theta), \varphi_g^B(\theta) < \theta$.

Consider the range $\theta \in [0, \overline{\theta}]$. $\varphi_b^N(\theta), \varphi_g^B(\theta) < \overline{\theta}$ for this range. Thus $v(\theta_N) = v(\theta_B) = 0$.

Also, there exists a $\theta' < \overline{\theta}$ such that $\varphi_g^G(\theta') = \overline{\theta}$.

Notice that, there does not exist $c > 0$ such that the auditor puts effort for $\theta \in [0, \theta')$.

Now, as $\theta \to 1$, $\varphi_g^G(\theta)$, $\varphi_b^N(\theta), \varphi_g^B(\theta) \to 1$ which implies that the right hand side of the above inequality goes to zero. Thus there exists a $\theta^* < 1$ such that for all $\theta > \theta^*$ right hand side of inequality (4) is decreasing in θ.

Define, $\overline{c} = min_{\theta \in [\overline{\theta}, \theta^*]}[p\epsilon[v(\varphi_g^G(\theta)) - v(\varphi_b^N(\theta))] + (1-p)(\alpha - \alpha')[v(\varphi_b^N(\theta)) - v(\varphi_g^B(\theta))]]$.

Fix $c \leq \overline{c}$.

There exists $\theta'' \in [\theta', \overline{\theta}]$ such that $\gamma = 1$ is optimal for the auditor for $\theta \in [\theta'', \overline{\theta}]$. This is because $v(\theta)$ is increasing.

Define, $\theta^1 = max\{\underline{\theta}, \theta''\}$.

Now, with $\gamma = 1$, there exists a threshold $\hat{\theta} > \overline{\theta}$, beyond which putting effort is not optimal for the auditor. Beyond this threshold c is strictly greater than $p\epsilon[v(\theta_G) - v(\theta_N)] + (1-p)(\alpha - \alpha')[v(\theta_N) - v(\theta_B)]$.

Now if the investor believes that the informative auditor does not put effort then there exists another threshold $\widetilde{\theta}$ such that the auditor has no incentive to be diligent if $\theta > \widetilde{\theta}$.

From Lemma 3 we know that, given θ, $p\epsilon(\varphi_g^G(\theta) - \varphi_b^N(\theta)) + (1-p)(\alpha - \alpha')(\varphi_b^N(\theta) - \varphi_g^B(\theta))$ is increasing in γ. Thus $\widetilde{\theta} < \hat{\theta}$.

θ^2 can take any value between $\widetilde{\theta}$ and $\hat{\theta}$ and the threshold equilibrium holds.

Therefore there exists a continuum of thresholds $\theta^2 \in [\widetilde{\theta}, \hat{\theta}]$ such that the strategy described in Proposition 1 is an equilibrium. ■

Proposition 1 crucially depends on the fact that there exists a range of θ where the posterior exceeds $\overline{p}$ even if the informative auditor does not exert due diligence. The informative auditor can shirk in that range of reputation only if three conditions are satisfied. First, the investor invests even when she believes that the auditor is not diligent. Second, following a mistake, reputation does not fall below $\overline{\theta}$, which is the threshold below which the auditor is not hired in period 2. If reputation does not fall below $\overline{\theta}$ the auditor continues to operate in the market even after making a mistake. Third, effort is costly, i.e. the difference between future payoff generated by high effort and low effort is smaller than the cost of putting effort.

The auditor meets a firm with a good project with probability p. By expending effort the auditor improves his chances of moving to a higher level reputation $\varphi_g^G(\theta)$ instead of moving to a lower level of reputation $\varphi_b^N(\theta)$. The gains from effort are captured by the difference between the expected capital raised at $\varphi_g^G(\theta)$ and $\varphi_b^N(\theta)$. The auditor meets a firm with a bad project with probability $1-p$. By expending effort the auditor reduces his chances of moving to a even lower level of reputation $\varphi_g^B(\theta)$. Gains from reputation is captured by the difference between the fee the auditor can charge by moving to the level of reputation $\varphi_b^N(\theta)$ and the fee he can charge if he moves to a lower level of reputation $\varphi_g^B(\theta)$ by passing a bad firm as good. The auditor is hired in the second period, only if his reputation is above $\overline{\theta}$. Thus for any reputation θ below $\overline{\theta}$, from where $\overline{\theta}$ can not be reached, the auditor is not diligent.

In equilibrium, for the auditor to be diligent, the difference between the value of choosing high effort and the value of choosing low effort must exceed the cost of choosing high effort. Suppose cost of expending effort is small. If the auditor is believed to be non-diligent, the value functions corresponding to high effort and low effort approach each other as $\theta \to 1$. This is because the values diverge only through the effect of current outcome on future posteriors. Current outcomes have very little effect on future posteriors if the market is quite sure of the auditor's type. Thus for posteriors close to unity, if the market believes that the auditor is not diligent, no matter how small the cost is, the gains from being diligent is surpassed by the cost of putting effort which gives rise to the threshold equilibria.

Proposition 1 formally shows that even when cost of effort is small, reputation concerns fail to solve the moral hazard problem under imperfect monitoring when reputation of the auditor is high. However, a high effort equilibrium, in which the auditor is diligent whenever he is hired, can be sustained with small costs under a more restrictive and non-generic assumption of perfect monitoring.

Perfect Monitoring: With the error structure being perfect monitoring, the informative auditor can perfectly distinguish between a good project and a bad project when he chooses to be diligent. The "informative" auditor's informativeness is still captured by the parameters α and ϵ, where α is the probability that a bad project is given the signal g and ϵ is the probability that a good project is given b. The informative auditor can improve precision by being diligent for which he has to bear a small cost $c > 0$. By being diligent the auditor passes a good project as good and a bad project as bad with probability 1.

Proposition 2: *There exists $\bar{c} > 0$ such that for all $c \leq \bar{c}$ there exist $\theta^1 \in [0, \bar{\theta}]$ so that the following pure strategy profile constitutes an equilibrium. At t=2, the auditor is never diligent. The auditor posts the fee $P(\theta) = \theta[p(1-\epsilon) + (1-p)\alpha] + (1-\theta)p$ and is hired only if $\theta \geq \bar{\theta}$. The investor invests only if $\theta \geq \bar{\theta}$ and $s = g$. At t=1, the auditor is not diligent, i.e. $\gamma(\theta) = 0$ for $\theta \in [0, \theta^1)$. $\gamma(\theta) = 1$ for $\theta \in [\theta^1, 1)$ and $\gamma(\theta) = 0$ for $\theta = 1$. The investor invests only if $\theta \geq \theta^1$ and $s = g$.*

Proof:

At $t = 2$, if the auditor is hired the firm pays him first and then the auditor decides whether to be diligent. His decision in that period does not affect his payoff in period 2. Thus, not putting effort in period 2 is a dominant strategy for the auditor. Now at $t = 2$, $P(\theta) = v(\theta)$, that is, the auditor extracts all the rents.

At $t = 1$, suppose the auditor is diligent. With probability p the informative auditor meets a good firm and moves to a higher level of reputation, say to $\theta_G = \varphi_g^G(\theta)$. With probability $(1-p)$ he meets a bad firm and receives signal $s = b$ with probability 1. No investment takes place and reputation is revised to $\theta_N = \varphi_b^N(\theta)$.

Therefore, the expected payoff by exerting effort in period 1 is given by, $pv(\theta_G) + (1-p)v(\theta_N) - c$.

Now if the auditor does not put effort in period 1, he moves to a higher level of reputation θ_G with probability $p(1-\epsilon)$. With probability ϵ he makes a mistake and receives $s = b$ which is followed by no investment and reputation falls to θ_N. With probability $(1-p)$ he meets a bad firm and receives signal $s = b$ with probability $(1-\alpha)$. No investment takes place and reputation is revised to θ_N. With probability α the auditor makes a mistake and receives $s = g$. The investor invests and the true quality of the project is revealed. This pushes reputation down to $\theta_B = \varphi_g^B(\theta)$.

Therefore, the expected payoff by not exerting effort in period 1 is given by, $p[(1-\epsilon)v(\theta_G) + \epsilon v(\theta_N)] + (1-p)[(1-\alpha)v(\theta_N) + \alpha v(\theta_B)]$.

The auditor will put effort in period 1 if,

$$c \leq p\epsilon[v(\theta_G) - v(\theta_N)] + (1-p)\alpha[v(\theta_N) - v(\theta_B)] \tag{5}$$

$$= [(1-p)\alpha - p\epsilon][p\epsilon(\theta_G - \theta_N) + (1-p)\alpha(\theta_N - \theta_B)] \;\; if \; \theta_N \geq \overline{\theta}$$

Consider the range $\theta \in [0, \overline{\theta}]$. $\varphi_b^N(\theta), \varphi_g^B(\theta) < \overline{\theta}$ for this range. Thus $v(\theta_N) = v(\theta_B) = 0$.

Also, there exists a $\theta' < \overline{\theta}$ such that $\varphi_g^G(\theta') = \overline{\theta}$.

Notice that, there does not exist $c > 0$ such that the auditor puts effort for $\theta \in [0, \theta')$.

Also notice that with $\hat{\gamma} = 1$ and $\alpha' = 0$, $\varphi_g^B(\theta) = \theta_B = 0$.

Now, as $\theta \to 1$, right hand side of the above inequality goes to $(1-p)\alpha[(1-p)\alpha - p\epsilon] > 0$.

Also, $p\epsilon[v(\theta_G) - v(\theta_N)] + (1-p)\alpha[v(\theta_N) - v(\theta_B)]$ is increasing in θ for $\theta \in [\theta', 1)$

Define $\overline{c} = p\epsilon v(\varphi_g^G(\overline{\theta}))$.

Fix $c \leq \overline{c}$.

There exists $\theta'' \in [\theta', \overline{\theta}]$ such that $\gamma = 1$ is optimal for the auditor for $\theta \in [\theta'', \overline{\theta}]$. This is because $v(\theta)$ is increasing.

Define, $\theta^1 = max\{\underline{\theta}, \theta''\}$.

Therefore for $c \leq \overline{c}$, the auditor has incentive to be diligent whenever hired and the "high effort" equilibrium holds. ■

The auditor meets a firm with a good project with probability p. In this event, gains from reputation approaches zero as $\theta \to 1$. Now the auditor meets a firm with a bad project with probability $1-p$ in which event, reputation gains may not diminish as θ approaches 1. This is sustained by the market belief that the informative auditor never makes a mistake. If he chooses to be non diligent, there is always a positive probability of making a mistake and moving to a lower reputation $\varphi_g^B(\theta)$. If the firm and the investor believes that the informed auditor never makes a mistake then $\varphi_g^B(\theta) = 0$. Therefore the difference between the expected capital raised at $\varphi_g^G(\theta)$ and $\varphi_b^N(\theta)$ is positive for values of θ for which the expected capital raised at $\varphi_b^N(\theta)$ is positive. As long as the expected cost of this mistake is higher than the cost of being diligent, the informative auditor will choose to be diligent. The mere fear of losing the market does not allow him to be lazy. At $\theta = 1$, learning stops and the optimal action for the auditor is not to exert diligence at that level of reputation. However, $\theta = 1$ is never reached if reputation in the first period is strictly less than 1.

The pure strategy equilibrium of Proposition 2 is the high effort equilibrium one might be interested in and this equilibrium arises only under the restrictive assumption of perfect monitoring coupled with a particular way of belief revision. However it is not the only pure strategy equilibrium in the perfect monitoring framework. There exists a continuum of threshold equilibria as described in Proposition 1. Equilibria with higher values of θ^2 are more efficient in the sense that they support high effort over a larger range of reputation.

5 Competition among auditors

Our analysis so far assumed monopoly of the auditor and no competition from other auditors in this market was assumed in the model. We now shift our attention to a duopoly market with two competing auditors, one investor, and one firm. The new model keeps the key aspects of the monopoly framework presented in section 3 and gives rise to similar threshold equilibria as described in Proposition 1.

Consider two identical auditors, auditor1 and auditor 2 with the same reputation θ. Each of these auditors can be informative or uninformative. In each period, the firm must hire only one auditor. The analysis in this section does not focus on report(rating) shopping aspect of the issuer pays markets. The focus of the analysis on the other hand is to capture the effect of competition on reputation incentives when auditors compete for clients.

Timeline: The sequence of events is as follows. At the beginning of period 1, the auditor who has been hired by the firm decides whether to be diligent by paying the cost c and acquires information about the project quality. He receives signal $s \in \{g, b\}$ depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest. If the investor invests her entire wealth, then the true project quality is revealed at the end of period 1. The investor does not observe project quality if no investment takes place. In period 2 the firm and the investor updates their belief about the auditor's type using Bayes' rule.

At the beginning of period 2, the investor and the firm believes that the auditors' are informative with probability θ and θ' respectively. The auditors post prices P and P' and the firm decides which auditor to hire. The hired auditor receives his fee upfront. The hired auditor decides whether to be diligent by paying the cost c and acquires information about the project quality. He receives signal $s \in \{g, b\}$ depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest.

Strategies and beliefs: A stationary strategy for the auditor is a pair (P, γ), i.e. , a fee and a choice of effort. Formally, $\gamma : [0,1] \times [0,1] \rightarrow [0,1]$, where $\gamma(\theta, \theta')$ is the probability that the hired auditor is diligent in period 1 which is a function of his own reputation and his rival's reputation. Similarly, $P : [0,1] \times [0,1] \rightarrow \mathbb{R}$ gives the fee an auditor posts in period 2.

A firm's strategy is a hiring function h from $\mathbb{R} \times \mathbb{R} \times [0,1] \times [0,1] \rightarrow \{0,1,2\}$, where $h = 0$ implies the firm does not hire any auditor, $h = 1$ implies that auditor 1 is hired and $h = 2$ implies that auditor 2 is hired.

The investor's investment strategy $a^* : [0,1] \times \{g, b, \phi\} \rightarrow [0,1]$ depicts how much the investor invests from a wealth w as a function of reputation of the hired auditor and the observed signal s.

The belief function $\pi : [0,1] \times \{g, b, \phi\} \rightarrow [0,1]$ gives the probability that the project is good given signal s and $\pi(\theta, s)$ is calculated using Bayes' rule.

If the "informative" auditor is diligent with probability $\gamma(\theta)$, then the posterior beliefs are updated using (1)-(3)

The expected capital raised by the firm for an auditor's certification is:

$$v(\theta) = \theta[p\{\hat{\gamma}(\theta) + (1 - \hat{\gamma}(\theta))(1 - \epsilon)\} + (1 - p)\{\hat{\gamma}(\theta)\alpha' + (1 - \hat{\gamma}(\theta))\alpha\}] + (1 - \theta)p$$

where, θ is the auditor's reputation and $\hat{\gamma}(\theta)$ is the belief about the auditor's choice of effort.

Definition: Equilibrium consists of a hiring strategy h by the firm, a choice of effort γ and a fee P by an auditor, an investment strategy a^* by the investor, a posterior function π and an updating rule φ such that,

1. h is optimal for the firm.
2. γ maximizes expected lifetime payoff for the auditor.
3. P maximizes period 2 payoff.
4. a^* is optimal for the investor.
5. π is obtained using Bayes' rule.
6. φ satisfies (1)-(3).

Proposition 3: *There exists $\bar{c} > 0$ such that for all $c \leq \bar{c}$ there exist $\theta^1 \in [0, \overline{\theta}]$ and $\theta^2 \in [\overline{\theta}, 1)$ so that the following pure strategy profile constitutes an equilibrium.*

At t=2, an auditor is never diligent. An auditor whose reputation is θ and whose rival's reputation is θ' posts a fee

$$P(\theta, \theta') = \begin{cases} max\{0, (\theta - \theta')[(1 - p)\alpha - p\epsilon]\} & if\ \theta' \geq \overline{\theta} \\ v(\theta) & otherwise \end{cases}$$

An auditor is hired only if $\theta \geq \overline{\theta}$ and expected capital raised by hiring that auditor is higher than that of his rival. The investor invests only if $\theta \geq \overline{\theta}$ and $s = g$.

At t=1, the auditor is not diligent, i.e. $\gamma(\theta) = 0$ for $\theta \in [0, \theta^1)$. $\gamma(\theta) = 1$ for $\theta \in [\theta^1, \theta^2)$ and $\gamma(\theta) = 0$ for $\theta \in [\theta^2, 1]$. The investor invests only if $\theta \geq \theta^1$ and $s = g$.

Proof:

At $t = 2$, if the auditor is hired the firm pays him first and then the auditor decides whether to be diligent. His decision in that period does not affect his payoff in period 2. Thus, not putting effort in period 2 is a dominant strategy for the auditor. Now $P(\theta) = max\{0, v(\theta) - v(\theta')\}$ at $t = 2$, that is, the hired auditor can post a positive fee only if his reputation is higher than his rival's reputation. If the auditor with reputation θ posts any price above $P(\theta)$, his rival can cut price and get hired by the firm.

Now, at $t = 1$, suppose the auditor exerts due diligence. With probability p the informative auditor meets a good firm and moves to a higher level of reputation, say to $\theta_G = \varphi_g^G(\theta)$. With probability $(1 - p)$ he meets a bad firm and receives signal $s = b$ with probability $(1 - \alpha')$. No investment takes place and reputation is revised to $\theta_N = \varphi_b^N(\theta)$. With probability α' the auditor makes a mistake and receives $s = g$. The investor invests and the true quality of the project is revealed. This pushes reputation down to $\theta_B = \varphi_g^B(\theta)$.

Therefore, the expected payoff by exerting effort in period 1 is given by, $pP(\theta_G)+(1-p)[\alpha' P(\theta_B)+ (1 - \alpha')P(\theta_N)] - c$.

If the auditor does not put effort in period 1, he moves to a higher level of reputation θ_G with probability $p(1 - \epsilon)$. With probability ϵ he makes a mistake and receives $s = b$ which is followed by no investment and reputation falls to θ_N. With probability $(1-p)$ he meets a bad firm and receives signal $s = b$ with probability $(1 - \alpha)$. No investment takes place and reputation is revised to θ_N. With probability α the auditor makes a mistake and receives $s = g$. The investor invests and the true quality of the project is revealed. This pushes reputation down to θ_B.

Therefore, the expected payoff by not exerting effort in period 1 is given by, $p[(1 - \epsilon)P(\theta_G) + \epsilon P(\theta_N)] + (1 - p)[\alpha P(\theta_B) + (1 - \alpha)P(\theta_N)]$.

The auditor will put effort in period 1 if,

$$c \leq p\epsilon[P(\theta_G) - P(\theta_N)] + (1 - p)(\alpha - \alpha')[P(\theta_N) - P(\theta_B)] \tag{6}$$

Since, from Lemma 1 we know that, $\varphi_g^G(\theta) > \theta$ and $\varphi_b^N(\theta), \varphi_g^B(\theta) < \theta$, $P(\theta_N) = P(\theta_B) = 0$ and

$$p\epsilon[P(\theta_G) - P(\theta_N)] + (1 - p)(\alpha - \alpha')[P(\theta_N) - P(\theta_B)] = p\epsilon[v(\theta_G) - v(\theta)]$$

Now,

$$p\epsilon[v(\theta_G) - v(\theta)] = p\epsilon(\theta_G - \theta)[(1-p)\alpha - p\epsilon] \;\; if\; \theta \geq \overline{\theta}.$$

Consider the range $\theta \in [0, \overline{\theta}]$. $\varphi_b^N(\theta), \varphi_g^B(\theta) < \overline{\theta}$ for this range.

There exists a $\breve{\theta} < \overline{\theta}$ such that $\varphi_g^G(\breve{\theta}) = \overline{\theta}$.

Notice that, there does not exist $c > 0$ such that the auditor puts effort for $\theta \in [0, \breve{\theta})$.

Also, as $\theta \rightarrow 1$, $\varphi_g^G(\theta) \rightarrow 1$ which implies that the right hand side of the above inequality goes to zero. Thus there exists a $\theta^{**} < 1$ such that for all $\theta > \theta^{**}$ right hand side of inequality (6) is decreasing in θ.

Define, $\bar{c} = min_{\theta \in [\overline{\theta}, \theta^{**}]}[p\epsilon[v(\varphi_g^G(\theta)) - v(\theta)]]$.

Fix $c \leq \bar{c}$.

There exists $\theta'' \in [\breve{\theta}, \overline{\theta}]$ such that $\gamma = 1$ is optimal for the auditor for $\theta \in [\theta'', \overline{\theta}]$. This is because $v(\theta)$ is increasing.

Define, $\theta^1 = max\{\underline{\theta}, \theta''\}$.

Now, there exists a threshold $\hat{\theta} > \overline{\theta}$, above which putting effort is not optimal for the auditor when $\hat{\gamma} = 1$.

Also if the investor believes that the informative auditor does not put effort then there exists another threshold $\widetilde{\theta}$ such that the auditor has no incentive to be diligent if $\theta > \widetilde{\theta}$.

From Lemma 3 we know that, given θ, $\varphi_g^G(\theta)$ is increasing in γ. Thus $\widetilde{\theta} < \hat{\theta}$.

θ^2 can take any value between $\widetilde{\theta}$ and $\hat{\theta}$ and the threshold equilibrium holds.

Therefore there exists a continuum of thresholds $\theta^2 \in [\widetilde{\theta}, \hat{\theta}]$ such that the strategy described in Proposition 1 is an equilibrium. ■

Proposition 3 characterizes the equilibria in a duopoly set up. Notice that the high effort equilibrium does not exist in the duopoly set up. No matter how small cost of effort is competition leads to threshold equilibria as described in Proposition 3. The next proposition addresses our second research question: does competition aid or inhibit reputation building behavior?

Consider the range of c for which the "threshold" equilibria as described in Proposition 1 and Proposition 3 holds for both monopoly and duopoly. The following proposition shows for any such c the range of reputation for which high effort can be sustained under monopoly is strictly larger than the range for which high effort can be sustained under duopoly. Define,

$c^M = min_{\theta \in [\overline{\theta}, \theta^*]}[p\epsilon[v(\varphi_g^G(\theta)) - v(\varphi_b^N(\theta))] + (1-p)(\alpha - \alpha')[v(\varphi_b^N(\theta)) - v(\varphi_g^B(\theta))]]$ and $c^D = min_{\theta \in [\overline{\theta}, \theta^{**}]}[p\epsilon[v(\varphi_g^G(\theta)) - v(\theta)]]$, where θ^* and θ^{**} are the way they have been defined in Proposition 1 and 3 respectively. Also define, $\overline{\overline{c}} = min\{c^M, c^D\}$.

Proposition 4: *For $c \leq \overline{\overline{c}}$, the range of reputation for which $\gamma = 1$ can be sustained in equilibrium is larger under monopoly.*

Proof:

Define, θ' such that $\varphi_g^G(\theta') = \overline{\theta}$.

Under monopoly, the auditor is diligent at θ' if $c \leq p\epsilon v(\overline{\theta})$.

Under duopoly, the auditor is diligent at θ' if $c \leq p\epsilon P(\overline{\theta})$.

Now, $P(\overline{\theta}) = v(\overline{\theta})$.

Define, $\theta^1 = max\{\underline{\theta}, \theta'\}$ as before.

Thus if $c \leq p\epsilon v(\overline{\theta})$ the auditor will be diligent for $[\theta^1, \overline{\theta}]$.

Now, consider the range $\theta \in [\overline{\theta}, 1]$ and suppose that the auditor is diligent.

Define, θ^N such that $\varphi_b^N(\theta^N) = \overline{\theta}$

Fix θ. From Proposition 3 we know that,

$$p\epsilon[P(\theta_G) - P(\theta_N)] + (1-p)(\alpha - \alpha')[P(\theta_N) - P(\theta_B)] = p\epsilon(\theta_G - \theta)[(1-p)\alpha - p\epsilon] \quad (7)$$

From Proposition 1, we know

$$p\epsilon[v(\theta_G) - v(\theta_N)] + (1-p)(\alpha - \alpha')[v(\theta_N) - v(\theta_B)] = [(1-p)\alpha - p\epsilon][p\epsilon(\theta_G - \theta_N) + (1-p)(\alpha - \alpha')(\theta_N - \theta_B)] \quad (8)$$

where, $\theta_G = \varphi_g^G(\theta)$, $\theta_N = \varphi_b^N(\theta)$ and $\theta_B = \varphi_g^B(\theta)$.

Now, $\varphi_b^N(\theta) < \theta$ and with $\gamma = 1$,

$\varphi_g^B(\theta) = \frac{\theta\alpha'}{\theta\alpha' + (1-\theta)p}$ and $\varphi_b^N(\theta) = \frac{\theta(1-\alpha')}{\theta(1-\alpha') + (1-\theta)}$.

From Lemma 2, we know that $\varphi_g^B(\theta) < \varphi_b^N(\theta)$.

Thus, the right hand side of equation (7) is equal to the right hand side of equation (8) for the range $\theta \in [\overline{\theta}, \theta^N)$ and is strictly greater for the range $[\theta^N, 1)$.

Therefore, the maximum range for which $\gamma = 1$ can be sustained in equilibrium is larger under monopoly. ■

Notice that, no matter what c is, reputation incentives for a monopolist is captured by $p\epsilon[v(\theta_G) - v(\theta_N)] + (1-p)(\alpha - \alpha')[v(\theta_N) - v(\theta_B)]$ while the reputation incentives for a duopolist is captured by $p\epsilon[v(\theta_G) - v(\theta)]$. Since, $\theta_B < \theta$ and $v(\theta)$ is increasing, gains from building reputation is always greater under monopoly. The above result is counter intuitive in some sense as reputation incentives are lowered under competition. The prevalent argument is, competition stimulates reputation building behavior as dissatisfied clients have the option of switching auditors. The very fear of losing customers to rivals is what drives reputation building behavior. Better reputation is typically associated with higher market shares and higher profits and competition is typically perceived to be a devise to sustain high effort.

The argument in support of the above result is as follows. In these kinds of markets competition among certification intermediaries is often manifested in the form of price competition and not quality competition. In order to attract clients, the fee the auditors charge in equilibrium are strictly lower than the fee they would charge under monopoly. Thus, competition may reduce expected future profit of an auditor and hence reduce incentives to put effort. In a setting as described in my model, an auditor can best reap the benefits generated by building reputation when he is not faced with price competition.

In equilibrium, for the auditor to be diligent, the difference between the value of choosing high effort and the value of choosing low effort must exceed the cost of choosing high effort. With probability p the auditor meets a firm with a good project. By expending effort the auditor improves his chances of moving to a higher level reputation $\varphi_g^G(\theta)$. By moving to a higher level of reputation the auditor can charge a positive fee which equals the difference between the expected capital the firm can raise by hiring the auditor and the expected capital the firm can raise by hiring his rival (whose reputation is θ). Under the monopoly set up the fee that the auditor can charge at $\varphi_g^G(\theta)$ equals the expected capital raised at that level of reputation. Gains from reputation under monopoly is captured by the difference between the fee the auditor can charge by moving to a higher level of reputation $\varphi_g^G(\theta)$ and the fee he can charge if he moves to a lower level of reputation $\varphi_b^N(\theta)$ by committing a mistake. The fee that the duopolist can charge if he moves to a lower level of reputation $\varphi_b^N(\theta)$ is zero, as his rival (who has a higher reputation θ) can cut price and still charge a positive fee. Thus the gains from building reputation is captured by the difference between the expected capital raised at $\varphi_g^G(\theta)$ and $\varphi_b^N(\theta)$ in a monopoly set up, while the gains from building reputation under duopoly is captured by the difference between the expected capital raised at $\varphi_g^G(\theta)$ and θ.

With probability $(1-p)$ the auditor meets a firm with a bad project. By expending effort the auditor reduces his chances of moving to a even lower level of reputation $\varphi_g^B(\theta)$. Gains from reputation under monopoly is captured by the difference between the fee the auditor can charge by moving to the level of reputation $\varphi_b^N(\theta)$ and the fee he can charge if he moves to a lower level of reputation $\varphi_g^B(\theta)$ by passing a bad firm as good. The fee that the duopolist can charge if he moves to any of these levels of reputations, $\varphi_b^N(\theta)$ or $\varphi_g^B(\theta)$ is zero, as his rival (who has a higher reputation θ) can cut price and still charge a positive fee. The gains from building reputation under duopoly disappears fully while the gains from building reputation under monopoly is still positive and is captured by the difference between the expected capital raised at $\varphi_b^N(\theta)$ and $\varphi_g^B(\theta)$. The presence of a rival leads to a reduction in fees that the auditors can charge and this in turn reduces their incentive to expend effort.

5.1 Firms have private information about their own type

Firms and investors may be asymmetrically informed about the quality of the project owned by the firm. This section deals with a situation when firms have private information about the project quality. auditors do not possess prior information about the project quality and can acquire information only if hired by the firm. Thus auditors can not price discriminate and have to post a single fee for the firms. The fee can be a pooling fee or a separating fee.

Suppose the firms are of two types, the high type or H firms and the low type or L firms. These types can be perceived as firms' virtual types, as there is still some uncertainty about the firms' true project quality. The true project quality can either be good or bad. A high type firm produces a good outcome with probability $p < \rho < 1$ and a bad outcome with probability $1-\rho$. A low type firm on the other hand produces a good outcome with probability $\rho' < p$ and a bad outcome with probability $1-\rho'$. The firm has private information about its own type. However the firm does not know the true quality of the project it owns. Whether the project is good or bad is revealed after the investor invests in it.

In period 2, the H firm's maximum willingness to pay for an auditor with reputation θ is

$$P_H(\theta) = \theta[\rho(1-\epsilon) + (1-\rho)\alpha] + (1-\theta)p$$

The L firm's maximum willingness to pay for an auditor with reputation θ on the other hand is

$$P_L(\theta) = \theta[\rho'(1-\epsilon) + (1-\rho')\alpha] + (1-\theta)p$$

Suppose that $\rho'(1-\epsilon) + (1-\rho')\alpha < p < \rho(1-\epsilon) + (1-\rho)\alpha$, i.e. a H firm receives the signal g from an informative auditor with a higher probability than it does from an uninformative auditor. The assumption also ensures that the L firm receives the signal g from an informative auditor with a lower probability than it does from an uninformative auditor.[4] Notice that $P_H(\theta) > P_L(\theta)$ for $\theta \geq \overline{\theta}$. Therefore, P_L is the pooling price at which both the H firms and the L firms hire an auditor. An auditor can charge the separating price P_H in which event he is only hired by the H firms. The H firms hire an auditor as there is no other way they can signal their type and investors invest only if the signal is g. This is because there is still some uncertainty about the firms' type and the signal s is the only source of information to the investor. Now, at a particular θ, which fee gives higher revenue to an auditor depends on the proportion of high type firms in the economy. Let x be the probability of meeting a high type firm every period. Thus, $x\rho + (1-x)\rho' = p \Longrightarrow x = \frac{p-\rho'}{\rho-\rho'}$. Now, revenue of an auditor from posting the separating price is $x\theta[\rho(1-\epsilon)+(1-\rho)\alpha]+x(1-\theta)p$, which is increasing in θ. Revenue from posting the pooling price on the other hand is $\theta[\rho'(1-\epsilon)+(1-\rho')\alpha]+(1-\theta)p$ which is decreasing in θ. For $\theta \geq \overline{\theta}$, define $R(\theta)$ such that,

$$R(\theta) = max\{x\theta[\rho(1-\epsilon) + (1-\rho)\alpha] + x(1-\theta)p, \theta[\rho'(1-\epsilon) + (1-\rho')\alpha] + (1-\theta)p\}$$

Consider two identical auditors, auditor1 and auditor 2 with the same reputation θ. Each of these auditors can be informative or uninformative. Because of the presence of a rival the only fee an auditor can post in equilibrium is the pooling fee.

[4]This assumption is both plausible and innocuous. It is in place for the ease of exposition.

Timeline: The sequence of events is as follows. At the beginning of period 1 an auditor, who has been hired by the firm in period 1, decides whether to be diligent by paying the cost c and acquires information about the project quality. He receives signal $s \in \{g, b\}$ depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest. If the investor invests her entire wealth, then the true project quality is revealed at the end of period 1. The investor does not observe project quality if no investment takes place. In period 2 the firm and the investor updates their belief about the auditor's type using Bayes' rule.

At the beginning of period 2, the investor and the firm believes that the auditors are informative with probability θ and θ' respectively. The firm privately observes its own type. The auditors post prices P and P' and the firm decides which auditor to hire. The hired auditor receives his fee upfront. The hired auditor decides whether to be diligent by paying the cost c and acquires information about the project quality. He receives signal $s \in \{g, b\}$ depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest.

Strategies and beliefs: A stationary strategy for the auditor is a pair (P, γ), i.e. , a fee and a choice of effort. Formally, $\gamma : [0, 1] \times [0, 1] \to [0, 1]$, where $\gamma(\theta, \theta')$ is the probability that the hired auditor is diligent in period 1 as a function of his own reputation and his rival's reputation. Similarly, $P : [0, 1] \times [0, 1] \to \mathbb{R}$ gives the fee an auditor posts in period 2.

A firm's strategy is a hiring function h from $\mathbb{R} \times \mathbb{R} \times [0, 1] \times [0, 1] \times \{H, L\} \to \{0, 1, 2\}$, where $h = 0$ implies the firm does not hire any auditor, $h = 1$ implies that auditor 1 is hired and $h = 2$ implies that auditor 2 is hired.

The investor's investment strategy $a^* : [0, 1] \times \{g, b, \phi\} \to [0, 1]$ depicts how much the investor invests from a wealth w as a function of reputation of the hired auditor and the observed signal s.

The belief function $\pi : [0, 1] \times \{g, b, \phi\} \to [0, 1]$ gives the probability that the project is good given signal s and $\pi(\theta, s)$ is calculated using Bayes' rule.

If the "informative" auditor is diligent with probability $\gamma(\theta)$, then the posterior beliefs are updated using (1)-(3)

where, θ is the auditor's reputation and $\hat{\gamma}(\theta)$ is the belief about the auditor's choice of effort.

Definition: Equilibrium consists of a hiring strategy h by the firm, a choice of effort γ and a fee P by an auditor, an investment strategy a^* by the investor, a posterior function π and an updating rule φ such that,

1. h is optimal for the firm.

2. γ maximizes expected lifetime payoff for the auditor.

3. P maximizes period 2 payoff.

4. a^* is optimal for the investor.

5. π is obtained using Bayes' rule.

6. φ satisfies (1)-(3).

Proposition 5: *There exists $\bar{c} > 0$ such that for all $c \leq \bar{c}$ there exist $\theta^1 \in [0, \overline{\theta}]$ and $\theta^2 \in [\overline{\theta}, 1)$ so that the following pure strategy profile constitutes an equilibrium.*

At t=2, an auditor is never diligent. An auditor whose reputation is θ and whose rival's reputation is θ' posts a fee

$$P(\theta, \theta') = \begin{cases} max\{0, (\theta' - \theta)[\rho'(1-\epsilon) + (1-\rho')\alpha - p]\} & if\ \theta' \geq \overline{\theta} \\ R(\theta) & otherwise \end{cases}$$

An auditor is hired only if $\theta \geq \overline{\theta}$ and expected capital raised by hiring that auditor is higher than that of his rival. The investor invests only if $\theta \geq \overline{\theta}$ and $s = g$.

At t=1, the auditor is not diligent, i.e. $\gamma(\theta) = 0$ for $\theta \in [0, \theta^1)$. $\gamma(\theta) = 1$ for $\theta \in [\theta^1, \overline{\theta})$ and $\gamma(\theta) = 0$ for $\theta \in [\overline{\theta}, 1]$. The investor invests only if $\theta \geq \theta^1$ and $s = g$.

Proof:

At $t = 2$, if the auditor is hired the firm pays him first and then the auditor decides whether to be diligent. Thus, not putting effort in period 2 is a dominant strategy for the auditor.

Now consider the range $\theta \in [\overline{\theta}$,1]

If $\theta' < \overline{\theta}$ the auditor posts a monopoly price $R(\theta)$.

If $\theta' \geq \overline{\theta}$, the auditor faces his rival and $P(\theta) = max\{0, P_L(\theta') - P_L(\theta)\} = max\{0, (\theta' - \theta)[\rho'(1-\epsilon) + (1-\rho')\alpha - p]\}$. The hired auditor can post a positive fee only if his reputation is lower than his rival's reputation.

Now, at $t = 1$, suppose the auditor exerts due diligence.

Therefore, the expected payoff by exerting effort in period 1 is given by, $pP(\theta_G)+(1-p)[\alpha' P(\theta_B)+(1-\alpha')P(\theta_N)] - c$ where, $\theta_G = \varphi_g^G(\theta)$, $\theta_N = \varphi_b^N(\theta)$ and $\theta_B = \varphi_g^B(\theta)$.

Now if the auditor does not put effort in period 1, his expected payoff is given by, $p[(1-\epsilon)P(\theta_G)+\epsilon P(\theta_N)] + (1-p)[\alpha P(\theta_B) + (1-\alpha)P(\theta_N)]$.

The auditor will put effort in period 1 if,

$$c \leq p\epsilon[P(\theta_G) - P(\theta_N)] + (1-p)(\alpha - \alpha')[P(\theta_N) - P(\theta_B)] \tag{9}$$

Now define, $\hat{\theta}$ such that $\varphi_g^B(\hat{\theta}) = \overline{\theta}$ and consider $\theta \geq \hat{\theta}$.

Since, $\varphi_g^G(\theta) > \theta$ and $\varphi_g^B(\theta) < \varphi_b^N(\theta) < \theta$, $P(\theta_G) - P(\theta_N) < 0$ and $P(\theta_N) - P(\theta_B) < 0$

Therefore, for any $c > 0$ $\gamma = 1$ can not be sustained in equilibrium for $\theta \geq \hat{\theta}$.

Consider the range $\theta \in [0, \overline{\theta}]$. $\varphi_b^N(\theta), \varphi_g^B(\theta) < \overline{\theta}$ for this range.

Define $\breve{\theta} < \overline{\theta}$ such that $\varphi_g^G(\breve{\theta}) = \overline{\theta}$.

Notice that, there does not exist $c > 0$ such that the auditor puts effort for $\theta \in [0, \breve{\theta})$.

Let $g(\theta)$ be the equilibrium expected gain of an auditor by being diligent.

Let $\overline{P} = min_{\theta \in [\overline{\theta}, \hat{\theta}]} g(\theta)$.

Fix $c \leq \overline{P}$.

There exists $\theta'' \in [\breve{\theta}, \overline{\theta}]$ such that $\gamma = 1$ is optimal for the auditor for $\theta \in [\theta'', \overline{\theta}]$.

Define, $\theta^1 = max\{\underline{\theta}, \theta''\}$.

Set $\theta^2 = \hat{\theta}$ and this concludes the proof. ■

The presence of a rival makes it impossible for a duopolist to post any fee above the pooling fee. The return function that the duopolist takes into account is guided by the pooling fee which is decreasing in reputation. For a lower range of reputation close to θ^1 the gains from reputation is same as the gains from reputation of a monopolist. This is because the rival may drop out in the second period because of low reputation. Now let us focus on the range of reputation where an auditor faces his rival in period 2 with probability 1, that is, no auditor's reputation falls below $\overline{\theta}$. We are particularly interested in this range of reputation as we are interested in finding out the impact of competition on reputation building behavior. For this range the relevant return function is guided by the pooling fee which is a decreasing function of reputation.

The auditor meets a firm with a good project with probability p. By expending effort the auditor improves his chances of moving to a higher level reputation $\varphi_g^G(\theta)$. By moving to a higher level of reputation the auditor does not gain anything as the fee he can charge is zero. His rival (whose reputation is θ) on the other hand can charge a positive fee which equals the difference between the expected capital a L type firm can raise by hiring the rival and the expected capital a L type firm can raise by hiring the auditor. The fee that the duopolist can charge if he moves to a lower level of reputation $\varphi_b^N(\theta)$ is positive and equals the difference between the expected capital a L type firm can raise by hiring the auditor and the expected capital a L type firm can raise by hiring his rival (who has a higher reputation θ).

The auditor meets a firm with a bad project with probability $1 - p$. By expending effort the auditor reduces his chances of moving to a even lower level of reputation $\varphi_g^B(\theta)$. If he moves to a lower level of reputation $\varphi_g^B(\theta)$ by passing a bad firm as good the fee he can post equals the difference between the expected capital a L type firm can raise by hiring the auditor and the expected capital a L type firm can raise by hiring his rival (who has a higher reputation θ). This is higher than the fee he can charge if he moves to $\varphi_b^N(\theta)$ by not committing a mistake. The fee the duopolist can charge at $\varphi_b^N(\theta)$ is difference between the expected capital a L type firm can raise by hiring the auditor at $\varphi_b^N(\theta)$ and the expected capital a L type firm can raise by hiring his rival (who has a higher reputation θ).

Thus for the range of reputation where both auditors remain in the market in period 2 with probability 1, the hired auditor in period 1, has no incentive to put effort. The intuitive explanation for the result is as follows. The lower range of reputation where the rival drops out in period 2 the auditor acts like a monopolist and may find it profitable to post a separating fee in the next period. The range of reputation where the auditor faces his rival in period 2 for sure if he is in the market, but he himself has a higher probability of dropping out next period if he is not diligent, then he will have incentive to put effort. But when he faces his rival for sure and is also sure that he himself will not have to drop out for not being diligent, the auditor stops putting effort. When firms have private information about the project quality they own the auditors compete to attract clients and can never charge a fee above the pooling fee and end up attracting the bad clients too. The H firms pay much less than their maximum willingness to pay and pricing decision is solely driven by the maximum willingness to pay by the L type clients. This hampers reputation incentives by making the return function decreasing in reputation. When firms and investors are symmetrically informed, as cost of being diligent goes to zero the threshold beyond which effort can not be sustained in equilibrium approaches unity. However when firms have private information about the project quality, the duopolist stops putting effort beyond the threshold as described in Proposition 5. No matter how small the cost of being diligent is sustaining high effort beyond that threshold is not possible in equilibrium.

6 Discussion

This section provides discussions on issues that are closely related to these two audience markets. Also these are issues that are frequently discussed when it comes to the understanding of the certification intermediary markets. Subsection 6.1 and 6.2 talks about report contingent payments and issuer-pays vs. the investor pays model. The discussions help us understand these issues as well as the functioning of these markets in light of the model presented in section 3. In subsection 6.3 I present a model with infinite horizon and discuss why the results of my model are robust.

6.1 Report contingent payments

Bolton et al. (2008) suggested regulatory intervention requiring upfront payments for rating services (before CRAs propose a rating to the issuer) combined with mandatory disclosure of any rating produced by CRAs to substantially mitigate the conflicts of interest of both CRAs and issuers. In the audit market up front payment is a regular practice. This section discusses how upfront payment can mitigate conflicts of interest when it comes to the choice of effort by a auditor in an issuer pays market. Though upfront payment can substantially weaken adverse effects of conflict of interest it can not eliminate the moral hazard problem embedded in these two sided markets.

Consider the following report contingent payment structure. The firm pays the auditor only if the signal is g.

Timeline: The sequence of events is as follows. At the beginning of period 1 the auditor posts a price P and the firm decides whether to hire the auditor. If the auditor is hired he decides whether to be diligent by paying cost c and acquires information about the project quality. He receives signal $s \in \{g, b\}$ depending on his choice of effort and the signal is observed by the investor and the firm. The investor decides how much to invest. The firm pays P to the auditor if and only if $s = g$.

If the investor invests her entire wealth, then the true project quality is revealed at the end of period 1. The investor does not observe project quality if no investment takes place. In period 2 the firm and the investor updates their belief about the auditor's type using Bayes' rule. The auditor posts a price P and the firm decides whether to hire the auditor. If the auditor is hired, he decides whether to be diligent by paying the cost c and acquires information about the project quality. He receives signal $s \in \{g, b\}$ depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest. The firm pays P to the auditor if and only if $s = g$.

Analysis: In each period, an auditor who is a monopolist posts a fee $P = 1$, that is, the auditor extracts all the rent when $s = g$. Notice that, now the choice of effort not only affects expected future payoff, but also affects current payoff of the auditor. In the one shot game the informative auditor puts effort if $c \leq (1-p)(\alpha' - \alpha) + p\epsilon$. Suppose, $(1-p)(\alpha - \alpha') - p\epsilon > 0$, that is, the auditor has short term incentive to shirk. Therefore, in the second period, as there is no reputation concern the auditor will not put effort. In the first period, the auditor has to face a short term loss in order to have reputation gain in the second period. Now in the second period, the expected payoff of an informative auditor who knows his own type is independent of reputation as long as the auditor is hired by the firm. Therefore if the auditor is hired in the second period, $EP(\theta) = p(1-\epsilon) + (1-p)\alpha$ for $\theta \geq \overline{\theta}$. Also, there exists $\hat{\theta} \in (\overline{\theta}, 1)$ such that $\varphi_g^B(\hat{\theta}) = \overline{\theta}$. Hence, the auditor never exerts diligence in the first period for $\theta \geq \hat{\theta}$ no matter how small c is. On the other hand as $c \to 0$, the range of reputation for which high effort can be sustained in the first period approaches unity with a upfront payment structure.

In a duopoly set up with two auditors, one firm and one investor, the auditor with a higher net expected payoff is hired. In case of a tie the auditor who receives $s = g$ with a higher probability is hired by the firm. Now notice that in period 2, if the auditor with a higher reputation charges $P = 1$, the auditor with lower reputation can cut price and get hired by the firm. Thus, the price the higher reputation auditor must charge in period 2 must be such that the expected net gain from hiring the auditor with higher reputation is same as that of hiring the auditor with lower reputation. Hence the expected second period payoff of the auditor with a higher reputation is the difference between the probability of him generating g and the probability that the lower reputation auditor generates g. Thus report contingent payments generate the same reputation incentives as upfront payments in terms of expected payoff in period 2. However the upfront payment structure performs better than the report contingent payment structure by not allowing the choice of effort affect the current payoff of the auditor.

6.2 Issuer pays vs investor pays

First, it is important to explain what investor pays means in these kinds of two sided markets where issuer pays is the rule. Investors are the ones who gain the most from the information value of the opinion issued by the auditors. In a litigation free world, investors bear all the risks of investment. It is often difficult to identify investors as one entity for publicly held companies. However, if the investor is a single entity there are several ways in which the investor can enter into a contract with the auditor.

In an "investor pays" model the investor hires the auditor who acquires information about the firm the investor wants to invest in. The auditor can be of two types "informative" and "uninformative". The "informative" auditor's informativeness is captured by the parameters α and ϵ, where α is the probability that a bad project is given the signal g and ϵ is the probability that a good project is given b. Assume that $p\epsilon < (1-p)\alpha$, i.e. ex-ante the informative auditor is more likely to commit an error when the project quality is bad. Without loss of generality, suppose that $r_2 = -1$, that is the investor loses all her money if the project is bad. The investor's gain from the auditor being diligent is bounded above by $w[(1-p)(\alpha-\alpha')+p\epsilon r_1]$. Thus, whenever $c > w[(1-p)(\alpha-\alpha')+p\epsilon r_1]$ it is not socially optimal for the auditor to exert diligence. Consequently, the auditor is hired only at a high level of reputation that is for $\theta \geq \overline{\theta}$.

The one shot game: Suppose the auditor is "informative" and the type is public information. In an issuer pays model with upfront payment the auditor is never diligent. In the "issuer pays" model the firm hires the auditor as long as α and ϵ is such that $Pr(G|g) > \overline{p}$ and $Pr(G|b) < \overline{p}$. The investor invests her entire wealth whenever $s = g$.

However under the "investor pays" model there exists $\overline{c} > 0$ such that for $c \leq \overline{c}$ the investor can design a contract contingent on outcomes, that can provide incentives for the auditor to exert diligence. Let $y \in \{-1, 0, 1\}$ denote the outcome in one period. $y = 1$, if $s = g$ and the outcome is good. $y = 0$, if $s = b$ and no investment is made. $y = -1$, if $s = g$ and the outcome is bad. Let $T(y)$ be the payment to the auditor when outcome is y. Now consider the following contract, $T(1) = x'$, $T(0) = x$ and $T(-1) = 0$. For the auditor to exert diligence, the following condition must hold

$$px' + (1-p)(1-\alpha')x \geq p(1-\epsilon)x' + (1-p)(1-\alpha)x + p\epsilon x$$

That is,

$$p\epsilon(x' - x) + (1-p)(\alpha - \alpha')x \geq c$$

Any combination of x and x' that satisfies the above inequality will make the auditor expend effort even if his type is known to be informative.

Dynamic Contract: Now consider the situation where the type of the auditor is not known. The prior probability that the auditor is informative is given by $\theta_0 \geq \underline{\theta}$. At $\underline{\theta}$, the investor is indifferent between hiring and not hiring the auditor if the auditor exerts diligence.

Since, at $t = 2$ there exists a contract that can induce effort the state contingent contract at $t = 1$ can be of the following form. At $t = 1$, a state contingent contract is a map from $[0, 1] \times \{-1, 0, 1\}$ to $\mathbb{R} \times \Psi$ which specifies a transfer from the investor and a contract for period 2. Let Ψ be the set of incentive compatible and individually rational contracts at $t = 2$. To look for an efficient contract is not the purpose of this exercise. The objective of this discussion is to argue for the existence of contracts that are incentive compatible and individually rational. Consider the following contract. At t=1, the investor pays according to the transfers specified in the one shot game. Any outcome that pushes reputation θ below $\underline{\theta}$ is followed by the auditor not being hired at $t = 2$. Any outcome that keeps reputation θ above $\underline{\theta}$ is followed by the same state contingent transfers as the one shot game at $t = 2$. By offering the contract described above the investor can make sure that the auditor exerts effort in both periods for the range of reputation $\theta \in [\underline{\theta}, 1]$. Thus under investor pays model with state contingent contracts the range of reputation for which effort can be sustained in equilibrium is larger.

Now suppose there are frictions that prohibit the investor from entering into state contingent contract with the auditor. However the investor can pay the auditor upfront for the information the auditor acquires about the project owned by the firm. The auditor makes a take it or leave it offer by posting a fee. In this kind of set up where the return function of the auditor is increasing in reputation, reputation concern may fail to incentivize effort after a threshold reputation and lead to qualitatively similar outcomes as the issuer pays model.

6.3 Infinite horizon:

This section aims to provide some intuition for why the results of the two period model have similar implications as one would get under an infinite horizon model. This is an interesting extension of the existing two period model. However this model does not provide us with any new insights. Hence the following discussion abstains from providing a rigorous analysis. In this model, time proceeds in discrete steps, indexed by $t = 1, 2, ...$ and has an infinite horizon. Each period, emerges a new firm that can yield a good outcome or a bad outcome. The auditor is long lived and has a discount factor $\delta \in (0, 1)$.

Timeline: The sequence of events is as follows. At the beginning of period t, the investor and the firm believes that the auditor is informative with probability θ_t. The firm which does not know its own type decides whether to hire the auditor and if the firm decides to hire the auditor it makes its payments in advance and the payment equals its maximum willingness to pay for the auditor. The auditor then decides whether to be diligent by paying the cost c and acquires information about the project quality. He receives signal $s \in \{g, b\}$ depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest. If the investor invests her entire wealth, then the true quality of the project is revealed at the end of period t. reputation is revised using Bayes' rule.

Strategies and beliefs: A Markov strategy for an informative auditor at any t and θ_t is a map $\gamma_t : [0, 1] \to [0, 1]$. Where, γ_t is the probability of exerting due diligence. A firm's strategy is a hiring function h from $[0, 1] \to \{0, 1\}$, where $h = 0$ implies the firm does not hire the auditor and $h = 1$ implies the auditor is hired. The investor's investment strategy $a^* : [0, 1] \times \{g, b, \phi\} \to [0, 1]$ depicts how much the investor invests from a wealth w as a function of reputation and observed signal s.

The Markov belief function $\pi : [0,1] \times \{g, b, \phi\} \to [0,1]$ gives the probability that the project is good given signal s and $\pi(\theta, s)$ is calculated using Bayes' rule.

At the beginning of each period the investor and the firm observes a signal $s \in \{g, b, \phi\}$ and at the end of that period one of the following three outcomes is observed. A good(G) outcome is observed if a good project is financed, a bad(B) outcome is observed if a bad project is financed and no(N) outcome is observed if no investment takes place. At any t, the history of outcomes until period t, is denoted $y^t = (y_1, y_2, ..., y_t)$ and reputation of the auditor θ_t is common knowledge. Reputation at period $t+1$, $\theta_{t+1} = \theta_{t+1}(\theta_t, \hat{\gamma}_t, y^t)$ is a function of reputation at period t, the firm and the investor's common belief about the competent auditors' strategies and the history of outcomes until period t. θ_{t+1} is updated using Bayes' rule. $\varphi(\theta|s, i)$ or φ_s^i is the posterior probability that the auditor is "informative" given signal $s \in \{g, b\}$, outcome $i \in \{G, B, N\}$ and prior probability θ. If the "informative" auditor is diligent with probability $\gamma(\theta)$, then the posterior beliefs are calculated using (1)-(3).

A Markov perfect equilibrium for a given prior p consists of consists of a hiring strategy h by the firm, a choice of effort γ by the auditor, an investment strategy a^* by the investor, a posterior function π and an updating rule φ such that,

1. h is optimal for the firm.
2. γ maximizes expected lifetime payoff for the auditor.
3. a^* is optimal for the investor.
4. π is obtained using Bayes' rule.
5. φ satisfies (1)-(3).

Let $V(\theta)$ be the continuation payoff of the auditor at reputation θ. An auditor with reputation θ will put effort if,

$$c \leq \delta p \epsilon [V(\theta_G) - V(\theta_N)] + \delta(1-p)(\alpha - \alpha')[V(\theta_N) - V(\theta_B)] \tag{10}$$

where $\theta_G = \varphi_g^G(\theta)$, $\theta_N = \varphi_b^N(\theta)$ and $\theta_B = \varphi_g^B(\theta)$.

Notice that the continuation payoff $V(\theta)$ is bounded as $\delta < 1$. Clearly, under imperfect monitoring, as $\theta \to 1$ the revisions become small and θ_G, θ_N, $\theta_B \to 1$. Therefore, the right hand side of the above inequality approaches zero. Thus there exists a threshold reputation beyond which effort can not be sustained in equilibrium. However with perfect monitoring high effort can be sustained in equilibrium for the range of $\theta \in [\underline{\theta},1)$ for c small enough.

Now under the duopoly model, there are two infinitely lived auditors whose reputation at $t = 1$ is same. Notice that the per period return for the duopolists are lower than that of the monopolist. This is because of the presence of the rival. The duopolist whose rival is in his close vicinity can charge a lower price than the monopolist and reputation gains are also smaller. The lower per period payoffs leads to the flattening of the return function which in turn reduces reputation incentives.

7 Conclusion

This paper provides an analysis of the issuer pays markets with the help of a simple model of reputation. The analysis focuses on an environment with upfront payments and publicly observable signals and seeks answers to the following questions. Does even a well-functioning reputation mechanism provide enough incentives for the auditors to produce high quality certification? Does competition among auditors improve reputational incentives? These are questions concerning the fundamentals of audit market and other issuer pays markets.

I show that in a monopoly set up reputation may provide incentives to generate effort under certain circumstances, but not always. In a monopoly set up an auditor may have incentives to be non-diligent for very high levels of reputation even if the cost of diligence is small. The range of reputation where the market is quite sure about the auditor's type the auditor may have incentives to shirk. The high effort equilibrium where the auditor exerts diligence whenever he is hired, is a fragile equilibrium and it holds only under the restrictive assumption of perfect monitoring. The threshold equilibria on the other hand do not depend on the error structure and they hold under both perfect monitoring and imperfect monitoring.

The answer to the second question to the readers' surprise is a No. Competition among auditors does not necessarily improve reputation building behavior. Instead, competition may reduce the incentive to put effort by shrinking long term expected profit. In the two period duopoly model I show that the range of reputation for which diligence can be sustained (when costs are small) is larger under monopoly. Reputation incentives are further weakened in a duopoly set up when firms have private information about the quality of their projects.

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