Financial Disclosure and Managerial Incentives

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Abstract

We explore the interplay between disclosure and incentives for risk taking. A firm seeks to invest in a risky project over time and can choose whether to disclose performance on that project to a market of investors who provide capital. In general, more disclosure leads to more efficient risk taking, causing the firm to pick the risk-efficient project under full disclosure. These results hold when the firm installs a manager to execute projects on its behalf. However, the optimal solvency bonus can achieve efficiency even without full disclosure. Thus, disclosure and optimal contracts are substitutes rather than complements.

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1 Introduction

In the post-mortem of the Great Financial Crisis of 2007 to 2008, one of the well and commonly accepted causes was excessive risk taking of managers, especially in financial and mortgage companies. Academics, politicians, and commentators have offered various solutions to this, one of which is more corporate disclosure. Had companies more readily disclosed the riskiness of assets and therefore management's decisions, investors could price the equity and debt of the company more accurately, thereby affecting the manager's compensation, and ultimately the risk decision ex ante. In fact, short sellers like David Einhorn of Greenlight Capital explicitly found the disclosures of Lehman Brothers exceedingly opaque, which led to his decision to short the bank based on his pessimistic beliefs about the true quality of its assets. But is this argument correct? Is disclosure an effective means of constraining managerial risk taking?

Yes. We build an economic model that combines disclosure with managerial incentives. Managers can pick between risky and safe projects. Their firm can reveal a signal about this project choice to an outside investor, who then lends capital to the firm based on this disclosure. The model assumes that corporate investments occur over time and investors must infer managerial actions from disclosures of varying quality. The disclosure can be perfect, absent, or noisy. The rate at which the investor lends capital to the firm is a direct consequence of his knowledge and beliefs.

We first solve the benchmark model in which the investor knows perfectly that the manager's action. In this case, the manager's investment choice achieves first best, in that the manager only chooses the risky project if it is efficient for him to do so. Full disclosures are therefore the gold standard, and as there is no information asymmetry between the outside investor and the internal manager.

We next study the extreme case of no disclosure, in which the investor knows nothing when lending to the firm. This may seem extreme, given that companies disclose information in financial reports about managerial actions and intentions. But as the example of Lehman Brothers suggests, sometimes this disclosure is so opaque that investors can infer little from such statements and disclosures. Now the investor must form beliefs about the manager's action. These beliefs can ultimately be arbitrary because the investor has no information on which to condition these beliefs. We show that the investor softens his lending choice: he will offer a lower interest rate for the risky project and a higher interest for a safe project, reducing the spread between the two interest rates.

This occurs because the investor does not know with full confidence the riskiness of the project, and therefore mixes between these two states of the world, creating a weighted average of the interest rates. As such, there is a range of projects such that the manager will select the risky choice even though it is efficient for him to select the safe project. This is a direct consequence of the lack of disclosure and the investor's optimal response through his equilibrium interest rate offered to the firm.

We examine partial disclosure, in which the investor receives a noisy signal of the manager's action choice between risky and safe projects. This will still induce the manager to select some risky projects when it is efficient to select safe, but this range or projects is smaller than before, and therefore the inefficiency shrinks. This confirms our first contribution, namely, that disclosure does curtail excessive managerial risk taking.

We then explore how contracting can interact with the disclosure choice. The chief problem is that the manager is selecting risky projects when he should be selecting safe projects. We allow the firm to pay the manager a solvency bonus, which will reward the manager if the project succeeds and the firm stays solvent. We first solve for the level of solvency bonus that will restore first best, and show that this efficient bonus increases inversely with the level of disclosure. Said differently, when disclosure is low, the firm must provide a high bonus in order to induce the manager to select the safe project. Contracts can therefore act as a **substitute** instrument, counteracting the effect of poor disclosure.

Initial literature looked at risk disclosures and managerial incentives independent of each other. Key studies that focused only on risk disclosure concluded that transparent disclosures would clearly communicate business models and the key risks that arise from them (Banziger et Al 2012, Banziger et Al 2014, Beyer et Al 2010). This greater disclosure is associated with more efficient risk-taking and improved risk-return tradeoffs for investors (Goldstein & Sapra 2013, Hirtle 2015). Because there is no universal theory of disclosure, types of disclosure have been divided into various categories. In accounting, disclosures have sometimes been divided into categories such as association-based, discretionary-based, and efficiency-based disclosures (Verrecchia 2001), which are all different in their degrees of disclosure (Admati & Pfleiderer 2000). Other ways to categorize disclosures take into account the amount of information an investor knows about a project (Boot & Thakor 2001, Zhang 2012), which influences risk-taking by entrepreneurs and investors (Hughes & Pae 2004).

Researchers have also mentioned key arguments against disclosure: disclosure can be costly for firms, disclosure could cause a firm to lose its competitive advantage, and the lack of disclosure can be efficient in the right circumstances. With regard to cost, a firm may need to disclose information through a third party like an accounting firm, which requires an additional cost to the firm (Admati & Pfleiderer 2000, Hermalin & Weisback 2012). With regard to competitive advantage, while disclosure could convey future positive prospects of a firm to investors, it could also reveal strategic information to competitors (Darrough 1993, Baumann & Nier 2004). However, full disclosure helps the financial market in evaluating a firm's value more accurately (Darrough and Stoughton 1990) and, as discussed in this paper, leads to the most efficient risk-taking. Additionally, Madhavan made a key point when showing that large investors who place multiple trades can benefit from the absence of trade disclosure in a fragmented market, as can dealers who face less price competition than in a unified market (Madhavan 1995). Other researchers argue that there are environments in which requiring full disclosure may reduce welfare (Naik et Al 1999). This paper furthers this discussion by showing how risk-efficiency can be achieved even without full disclosure through an optimal solvency bonus.

Alternatively, there is also previous research focused solely on managerial risk-taking incentives. Foundational research on the conditions of risk, how it is used to predict the behavior of capital markets (Sharpe 1964), and how it affects rational decision rules for individuals was initially collected (Lintner 1965, Merton 1974). Later, other researchers expounded upon this and presented several ways in which investors could mitigate managerial risk-taking incentive problems stemming from self-interest (Bolton & Scharfstein 1990, Diamond 1991, FDIC 2014, Skinner 1994). Although alignment of incentives of management with the interests of investors is a crucial mechanism of corporate governance, risk-taking incentives are more effective in controlling risk-taking than legal regulations are (John & Qian 2003, John et Al 2000). While managerial performance evaluations are one tool that could be used to increase firm/project efficiency (Lambert 2001), equity-based incentives could cause managers to take highly inefficient risks (Tung 2011). New microeconomic theories of banking are the focus of much research on managerial incentives; these theories assert that different economic agents possess different pieces of information on relevant economic variables and will use this information for their own profit. This paradigm is useful for many areas of economic analysis (Freixas and Rochet 2008, Holmstrom & Tirole 1997), including other business sectors, which will be explored in this paper.

Later, others began to study public risk disclosures' affects on managerial risk-taking incentives (Cordella & Yeyati 1998), and they found that a firm's disclosure process reflects both the manager's obligation to disclose mandatory information and voluntary disclosure incentives (Heitzman, Wasley, Zimmerman 2010, Einhorn 2005). Managers have been found to provide higher quality disclosure before selling shares (Rogers 2008); this theory can be applied to how firms disclose risk to potential capital investors, which is examined in this paper. Moreover, managers with higher compensation incentives are far more likely to be associated with a higher degree of disclosure (Bansal et Al 2013). Further theory on how voluntary disclosures interact with incentives has emerged, relating it to game theory with the following central premise: any entity contemplating making a disclosure will disclose information that is favorable to the entity, and will not disclose information unfavorable to the entity (Dye 2001, Wagenhofer 1990). While these studies illustrate that there is a relationship between managerial incentives and firm risk disclosures, research is limited on how varying levels of disclosure (perfect, absent, or noisy disclosures) affect managerial incentives, which is what this paper explores.

Key research on contracts by Lambert shows that compensation contracts interact with models of incentive problems caused by moral hazard and adverse selection problems (Lambert 2001). Other research on contracting includes how the best contracts provide optimal risk-sharing (Diamond & Dybvig 2000), how contracts between agents should be contingent on the events in the information partitions of both agents (Townsend 1979), and how contracts affect incentives between two parties with asymmetric information (Harris & Raviv 1979, Singh 2003). The underlying assumption is that individuals respond to contracts that reward performance (Prendergast 1999), which could give managers the incentive to take excess risk. However, these studies do not consider how contracting can interact with the disclosure choice, and this paper aims to show that the solvency bonus returns the system to efficiency based on the degree of disclosure.

2 The Model

A firm seeks to invest in a risky project and requires capital from an outside creditor (investor). There are three risk-neutral players: the firm who installs a manager and borrows from the investor, the investor who lends to the firm in a competitive market, and the manager who picks the project. There are two types of projects: a risky (R)

project and a safe (S) project. Each project $i = R, S$ returns V_{Gi} with probability p_i and V_{Bi} with probability $1 - p_i$, so p_i is the probability of success. The firm raises capital I at interest rate r and must pay back $(1+r)I$ at the end of the game. Suppose $0 < V_{BR} \leq V_{BS} \leq I \leq V_{GS} \leq V_{GR}$ and $p_S \geq p_R$.

The actions of the manager are a_R for risky and a_S for safe. The manager chooses the project type with noise. The true state of the project is $\theta \in \{\theta_S, \theta_R\}$. Let $Pr(\theta_i|a_i) = 1 - \varepsilon$, where $\varepsilon > 0$ is small. This assumption is analogous to the standard assumption in agency models where the manager's performance measure is a distortion of his true effort choice. For example, the manager may want to implement the risky project, but for reasons outside of his control, the firm cannot execute the manager's desires and instead ends up implementing the safe project. There is residual uncertainty in the manager's actions that cannot be eliminated, resulting from his inability to control the output of the firm completely.[1]

The signal $\sigma \in \{S, R, \phi\}$ from the firm provides information to the investor on the performance of the project. If $\sigma = \phi$, then the firm does not disclose anything, which we call the no disclosure regime. Alternatively, the firm can disclose $\sigma = S$ or R as either safe or risky. We call this the disclosure regime and consider both full and partial disclosure. The timeline of the game runs as follows: The manager chooses between the risky project and the safe project. Investors observe signal $\sigma \in \{S, R, \phi\}$. The firm raises funds I at the interest rate r. Nature resolves uncertainty, and cash flows realize. Figure 2 illustrates the timeline.

If the project fails, the firm goes bankrupt. In that case, it returns V_{Bi} to the investor, who takes a loss, and the firm shuts down. The firm writes an output-sharing contract with the manager based on end of stage firm value. Specifically, the manager earns $\alpha \in (0,1)$ of firm value if the firm is solvent. Otherwise, he earns zero. The manager's compensation from action a_i is

$$Pay_i = \begin{cases} \alpha \left\{ V_{Gi} - (1+r)I \right\}, & \text{solvency,} \\ 0, & \text{bankruptcy.} \end{cases} \tag{1}$$

The manager is protected by limited liability since he receives no payment in bankruptcy. With unlimited liability, he would be responsible for losses of the firm, but this does not

[1]The assumption of noisy state space realization is primarily technical in nature, to ease the equilibrium analysis throughout the paper. Since ε can be arbitrarily small, we do not see this assumption as strong.

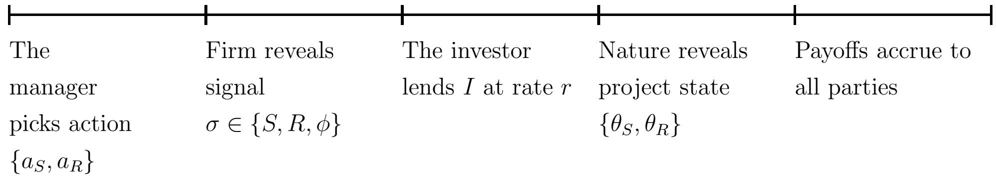

Figure 1: Timeline of the game.

fit actual practice. The net present value of a project is:

$$NPV_i = p_i V_{Gi} + (1 - p_i) V_{Bi} \,. \tag{2}$$

Efficiency requires selecting a_S if and only if $NPV_S \geq NPV_R$.

2.1 Full Disclosure

To fix ideas, suppose the signals that the firm releases to the investor are perfectly informative. Perfectly informative signals imply $Pr(\theta_i \,|\, \sigma = i) = 1$ for $i = S, R$. Figure 2 shows the game tree under a perfectly informative signal. After the manager chooses his action, the firm releases the signal, and the investor offers an interest rate to the firm. Because the signal has no noise, the investor can offer an interest rate that exactly matches the disclosure. We represent this with the variable r_σ, where σ is the signal that the investor observes. After offering the interest rate, Nature resolves uncertainty on success or failure, and the manager and investor collect their payoffs.

Observe that there is no asymmetric information because of the perfect information, and so the investor will know for sure whether he sits at the upper or lower branch of the game tree in Figure 2. Moreover, because of the perfect information, it is straightforward to calculate the sub-game perfect Nash equilibrium. To do so, simply take the expected value at each mode, rolling the game backwards. Doing this results in our first benchmark (all proofs are in the Appendix):

Proposition 1 *Full Disclosure is efficient.*

This should not be a surprise. Under full disclosure, all parties are symmetrically informed of the underlying uncertainty. The firm discloses the signal, and the investor perfectly tailors his interest rate to that disclosure, allowing the manager to make the efficient decision.

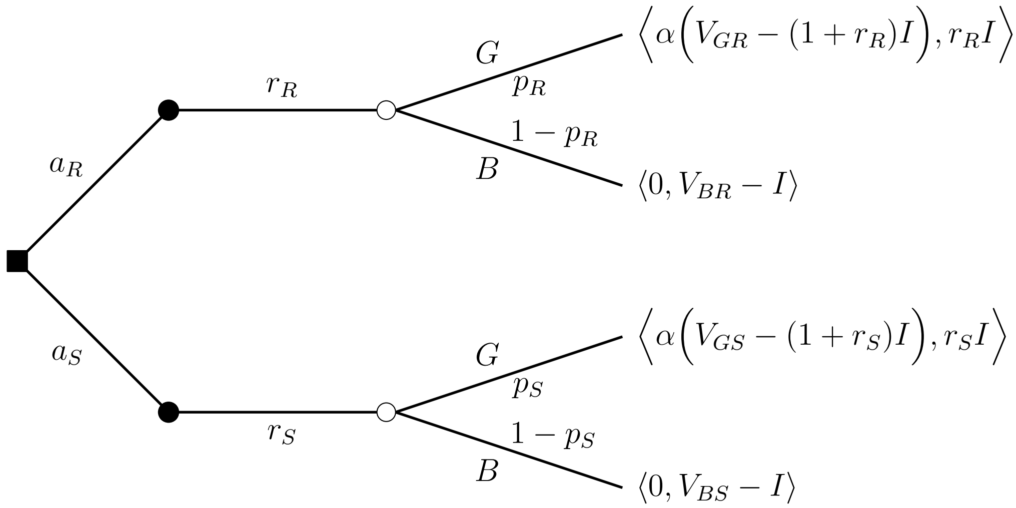

Figure 2: The Full Disclosure game. Squares represent moves by the manager. Dark circles represent moves by the investor. Hollow circles represent moves by Nature. The term in brackets represent the payoffs, first to the manager and second to the investor.

The interest rate charged by the investor in a competitive market would satisfy the zero profit condition $p_i(1 + r_i)I + (1 - p_i)V_{Bi} = I$, or

$$r_i = \frac{I - (1 - p_i)V_{Bi}}{p_i I} - 1. \tag{3}$$

The zero profit condition of the investor will determine the equilibrium interest rate. This zero profit investment stems from the competitive market in which investors supply capital to the firm. Notice that $r_R > r_S$. As expected, higher risk requires higher return. The investor will charge a higher interest rate for the more risky project since it must break even in equilibrium (because of the competitive market for capital).

2.2 No Disclosure

Now consider the no disclosure regime: the firm discloses nothing (i.e. discloses the null signal ϕ). As such, this will naturally lead to a game of incomplete information. We will look for a Perfect Bayesian Equilibrium where the investor will form beliefs about the manager's actions. Because there is no disclosure at all, he can form these beliefs however he likes.

Please see the game tree in Figure 3, which shows the extensive form of the game with incomplete information. After the manager chooses his action, the investor forms beliefs about the manager's action denoted by p_{ij}, the probability of signal $\sigma = j$ given action a_i. The investor forms these beliefs based on this signal, leading to the ultimate resolution of a good or bad project from nature.

Strategies, beliefs, and equilibrium concept: Suppose that $\mu = Pr(a = a_R)$ is the probability that the manager invests in the risky project and $\hat{\mu}$ is the investor's belief that $a = a_R$. A strategy for the manager is a choice of action a at the beginning of the first period given the disclosure regime. The second period interest rate r is determined by the investor depending on his belief $\hat{\mu}$. Now, r is a function of $\hat{\mu}$. In equilibrium, the manager must choose a_i such that his payoff at the end of the second period is maximized and beliefs coincide with actions.

Because the investor has no information on which to form his beliefs, we can examine any number of beliefs that can generate equilibrium behavior. To fix ideas, we will consider the extreme cases when the investor believes that the manager has chosen either a safe or risky project with certainty. First, consider the case when the investor believes that the manager chose the safe project ($\hat{\mu} = 0$). The market interest rate is $\hat{r_S}$ such that

$$\mathbb{E}_i\Big[(1 + \hat{r_S})p_i I + (1 - p_S)V_{Bi}\Big] = I\,, \tag{4}$$

where the expectation is taken over $i = \{S, R\}$ and weighted by probability ε and $1 - \varepsilon$, respectively. Now, suppose that the investor believes that the manager selects the risky project ($\hat{\mu} = 1$). The market interest rate is $\hat{r_R}$ such that

$$\mathbb{E}_i\Big[(1 + \hat{r_R})p_i I + (1 - p_i)V_{Bi}\Big] = I. \tag{5}$$

Simple computation of equations (3), (4), and (5) shows $r_S < \hat{r_S} < \hat{r_R} < r_R$. Thus, not only do risky projects earn higher return than safe projects, but the imputed interest rates of the investor, based on his beliefs, are less variable than the interest rates based on full disclosure. When the investor knows the disclosure, he will form the most confident estimate of the appropriate interest rate, either r_S or r_R, based on this certain knowledge. But if he must rely on his beliefs, which may be incorrect with small probability, this will shrink the range of possible interest rates that the investor will choose, thereby giving the rank-ordering $r_S < \hat{r}_S < \hat{r}_R < r_R$. In effect, the uncertainty in the game generated by the lack of disclosure softens the investor's action; it induces him to offer

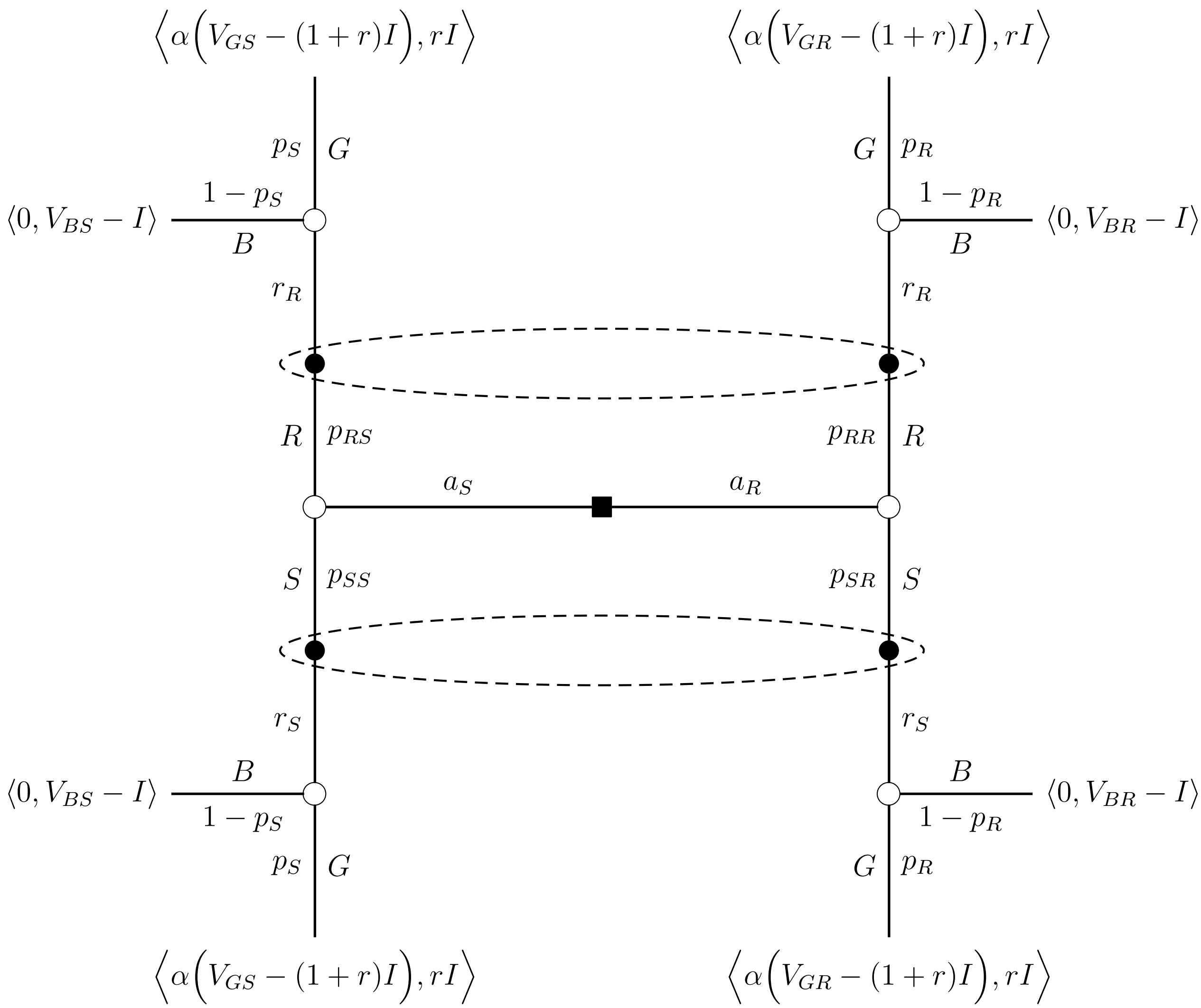

Figure 3: The Incomplete Disclosure game. Squares represent moves by the manager. Dark circles represent moves by the investor. Hollow circles represent moves by Nature. The term in brackets represent the payoffs, first to the manager and then to the investor. The dashed lines represent the information sets of the investor.

a higher interest rate for safe projects and a lower interest rate on risky projects. Given the investor's interest rate, we can now solve for the manager's optimal project choice.

Proposition 2 *Without disclosure, there exists the following pure strategy equilibria: the manager picks a_S if $NPV_S - NPV_R \geq p_S(r - r_S)I + p_R(r_R - r)I$, and a_R otherwise, for $r = r_S, r_R$ and for parameter values satisfying the following condition:*

$$NPV_S - NPV_R \in \Big(p_S(\hat{r_S} - r_S)I + p_R(r_R - \hat{r_S})I,\ p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I \Big). \quad (6)$$

The no-disclosure regime, therefore, generates inefficiency. The parameter values for which inefficiency occurs is given by (6). Before, full disclosure induced the manager to pick the safe project if and only if it was efficient to do so. But now, the manager will pick the safe project only if its NPV exceeds the efficient NPV threshold. The manager requires extra incentive to invest in the safe project because of the investor's poor information and arbitrary beliefs. Indeed, the investor's uncertainty softens the interest rate that he offers to the manager, thereby decreasing the spread between the risky interest rate and the safe interest rate. This decrease in spread induces the manager to accept risky projects more often. This is why the manager is more likely to adopt inefficiently risky projects. The region of projects in (6) are precisely those where the manager will select a risky project when it is efficient for him to select a safe project. Thus, the lack of disclosure creates incentives for the manager to take excess risk.

The intuition for this follows: The interest rate that the investor charges is a compensation for risk. As the riskiness of the project increases, the investor charges a higher interest rate to offset this higher risk. This interest rate is a cost to the firm since it drains the value of the firm. Since the manager is paid on firm value, it is also a cost to the manager. Without a higher interest rate for risky projects, the manager would be tempted to always choose the risky project. The risky interest rate dampens this incentive so that he can efficiently choose the right project. If the safe project is efficient, then the investor chooses a high interest rate (under full disclosure) in order both to compensate for risk as well as discourage the manager from always choosing the risky project.

In particular, it is the spread in interest rates (between the risky and safe projects) that prevents the manager from taking excessive risk. But when the lack of disclosure forces this spread to narrow, the dampening effect shrinks, and therefore, the manager is tempted to take more risk. Alternatively, a high spread modulates this temptation and keeps the manager's risk-taking incentives at bay. Finally, observe that the distortion in the manager's behavior is asymmetric. He is only induced to take excess risk, not excess safety. This occurs precisely because the disclosure softens the investor's action, reducing the range of interest rate. This reduction in the range directly stimulates extra risk.

Under the No Disclosure regime, the equilibrium investment decision is given by:

$$a = \begin{cases} a_S, & \text{if } NPV_S \geq NPV_R + p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I, \\ a_R, & \text{otherwise}. \end{cases} \tag{7}$$

But the above investment strategy is inefficient for the parameter range

$$NPV_S \in (NPV_R, NPV_R + p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I).$$

Thus, the value of the safe project must be sufficiently high in order to be efficient. Indeed, for projects that lie inside the range above, the manager will not select them even though it is efficient to do so. This is the manner in which disclosure itself generates inefficiency. The manager's action is a function of his information, which itself depends on the disclosure regime. It is precisely the lack of disclosure that generates the range of projects above and the consequent inefficiency.

2.3 Partial Disclosure

Now suppose that the firm releases only partial disclosure about the project choice to outside investors. Investors do not observe perfectly informative signals about the true state, so $Pr(\sigma = i \,|\, \theta = \theta_i) < 1$ for $i = S, R$. They infer the true state based on their belief about the manager's action, the observed signal, and their knowledge of the signal distribution. Let $p_{ij} = Pr(\sigma = i|\theta_j)$ be the probability distribution for the signals.

For the sake of simplicity, we assume that $p_{RR} = p_{SS} = \rho$, where $\rho \in (\frac{1}{2}, 1)$. Since the investors infer the true state based on their beliefs and the signal, the market interest rate now is a function of the signal σ and belief $\hat{\mu}$, so $r = r(\sigma, \hat{\mu})$.

Suppose $\hat{\mu} = 0$. We define $\bar{r_S}$ to be the market interest rate when $\sigma = S$ and $\bar{r_R}$ to be the market interest rate when $\sigma = R$. Therefore,

$$E(r_\sigma, \theta_S) = \rho\bar{r_S} + (1-\rho)\bar{r_R} \tag{8}$$

is the expected interest rate when the true state realization is θ_S. Similarly, when $\hat{\mu} = 1$, the market interest rate is $\tilde{r_S}$ with $\sigma = S$ and the market interest rate is $\tilde{r_R}$ with $\sigma = R$.

Therefore $E(r_\sigma, \theta_R) = \rho\tilde{r_S} + (1-\rho)\tilde{r_R}$. Having solved for the investor's equilibrium interest rate, we can now turn to the equilibrium in the partial disclosure game.

Proposition 3 *The following pure strategy profiles are equilibria in the Partial Disclosure regime. The manager picks* a_S *if* $NPV_S + p_S\Big(r_S - E(r_\sigma, \theta_S)\Big)I \geq NPV_R + p_R\Big(r_R - E(r_\sigma, \theta_R)\Big)I$*, and* a_R *otherwise, where*

$$NPV_S - NPV_R \in \Big[p_R\Big(r_R - (1-\rho)\bar{r_S} - \rho\bar{r_R}\Big)I - p_S\Big(r_S - \rho\bar{r_S} - (1-\rho)\bar{r_R}\Big)I\Big], \\ p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I - p_S\left(r_S - \rho\tilde{r_S} - (1-\rho)\tilde{r_R}\right)I. \qquad (9)$$

With partial disclosure, the investment strategy is such that there is inefficiency for the range

$$NPV_S \in (NPV_R,\, p_S\left(\rho\tilde{r_S} + (1-\rho)\tilde{r_R} - r_S\right)I + NPV_R - p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I).$$

As before, the value of a safe project must be sufficiently high in order for the manager to select it. Now, we still have disclosure leading to inefficiency since there is a range of safe projects that the manager will approve, but it is inefficient for him to do so. The partial disclosure still reduces the range of interest rates that the investor offers to the manager and firm. This follows from the same intuition as before, namely, that the investor does not precisely know the true state of the project. However, this range is smaller than under no disclosure.

Therefore, partial disclosure effectively reduces inefficiency since it shrinks the range of inefficient projects that will be approved by the manager.[2] The intuition follows. Recall that in the prior subsection, no disclosure shrinks the spread in interest rates between risky and safe projects relative to the full disclosure case. Shrinking the spread softens the investor's response and, thereby reduces the cost of risk-taking activities for the manager, which consequently increases his incentives to take risk. Partial disclosure has the same effect, but the interest rate spread does not decrease as much as the no disclosure case. Ultimately, partial disclosure is a compromise between full disclosure and no disclosure. It creates more inefficiency relative to full disclosure but not as much as no disclosure. This generates a strict ordering: the more the firm discloses, the less the inefficiency in project selection.

[2]Disclosure reduces inefficiency in the range of parameter values $NPV_S \in [p_S(\rho\tilde{r_S} + (1-\rho)\tilde{r_R} - r_S)I + NPV_R - p_R(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R})I,\ NPV_R + p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I]$.

3 Efficient Solvency Bonus

A natural question is whether there are contracting solutions to this inefficiency. Following the Coase theorem, absent transaction costs, parties should bargain to efficient outcomes through an appropriate set of contracts. Consistent with the literature on pay for performance, we can consider bonus payments that the firm structures with its manager in order to eliminate inefficient project selection. Imagine that the firm has available a solvency bonus that pays the manager for keeping the firm solvent. This bonus, represented by $\beta > 0$, could either take the form of an explicit payment contingent on the state of the world or could possibly be interpreted as inside debt whose value increases under solvency. Now suppose that the firm selects the manager's compensation β to maximize its own payoffs, consistent with most agency models. The firm's profit function is

$$\pi = \begin{cases} (1-\alpha)\left\{V_{Gi} - (1+r)I - \beta\right\}, & \text{solvency} \\ -k, & \text{bankruptcy.} \end{cases}$$

Observe that β drains firm value and so is clearly a cost to the firm. The manager's payoff function is

$$Pay_i = \begin{cases} \alpha\left\{V_{Gi} - (1+r)I - \beta\right\} + \beta, & \text{solvency} \\ 0, & \text{bankruptcy.} \end{cases}$$

Because the manager is paid on firm value, the smaller firm value from a positive β payment will make the manager worse off. But this payment goes directly to the manager, so that makes the manager better off. The second effect outweighs the first since the manager owns less than the entire firm ($\alpha < 1$). Therefore, this solvency-contingent bonus is certainly of value to the manager.

What is the level of bonus that would eliminate the inefficiency?

Proposition 4 *There exist efficient solvency bonus thresholds β^* and β^{**}:*

1. *If $\beta \geq \beta^*$ efficiency obtains without disclosure.*

2. *If $\beta \in (\beta^{**}, \beta^*)$ efficiency obtains with a combination of solvency contingent bonus and partial disclosure.*

3. *If $\beta < \beta^{**}$ there is inefficiency with partial disclosure.*

If the firm pays a sufficiently high bonus, it can completely reduce the inefficiency. But even with a bonus in a smaller range, a combination of partial disclosure and the bonus can still reduce inefficiency. But if the bonus is too small, then the disclosure and inefficiency still remain under partial disclosure. The thresholds β^* and β^{**} are detailed in the proof of the Proposition, but the intuition is straightforward.

The solvency bonus is a payment the firm makes for project success. Risky projects may generate higher value but are more likely to fail. Because of limited liability, the manager feels no downside penalty under bankruptcy and, therefore, has the incentive to take excessive risk if there is less than full disclosure. However, a bonus for project success can counteract this temptation to take a risk. Now, the manager receives an extra payoff only if the project succeeds, making him more likely to select the safer project because of its higher success probability. If the bonus is sufficiently high, this will completely overturn the manager's incentive to take excessive risk and return the system to efficiency.

Of course, this bonus may be limited given institutional constraints in compensation payouts or restrictions on the manager's ability to borrow and save over time. For example, this can occur if there is political outrage from paying large bonuses to managers, or the firm may voluntarily adopt a clawback compensation scheme that withholds large bonus payments to managers in one period and instead holds that bonus in escrow for the future, thereby aligning incentives over time. If there are such restrictions on compensation, then for an intermediate range of bonuses, the manager can still implement the efficient project choice with a combination of bonus and partial disclosure. The restricted compensation prevents the bonus by itself from counteracting the inefficiency from no disclosure, but the firm can supplement the bonus with partial disclosure that will together bring the system to efficiency. Finally, if the bonus is too small to be effective, even with partial disclosure, we still have inefficiency.

In this sense, contracts and disclosure are substitute instruments in reducing inefficiency. The firm can either compensate the manager to keep the firm solvent or disclose information to the marketplace, which will effectively induce more efficient managerial behavior. The level of inefficiency is a direct function of the level of disclosure and also the size of the bonus. Larger compensation payments reduce inefficiency, as does more disclosure. This speaks to the question posed by Leuz (2001), which mentions that the disclosure and contracting literatures have largely developed independent of each other. Yet firms use both instruments in their optimal design, and therefore, it behooves the researcher to consider both effects.

4 Optimal Solvency Bonus

The prior analysis only asks for the level of bonus that reduces inefficiency. Note that this may be different from the question of the firm's optimal choice of bonus. Moving to first-best may maximize total surplus, but it is not necessarily privately optimal for the firm. We now depart from our benchmark in this section to consider the optimal solvency bonus that the firm will set to maximize its own payoffs.

It remains to solve the optimal choice of β from the firm's perspective. As usual, first consider the full-disclosure case.

4.1 Full disclosure

When there is full disclosure, the manager selects a_S if and only if $NPV_S - NPV_R \geq 0$. However, under full disclosure and $\beta = 0$, the firm gains from a_S only when $[NPV_S - NPV_R] + (p_S - p_R)k \geq 0$. Observe that the firm loses money in bankruptcy, and therefore, the benefit of a safe project over a risky project must exceed the expected cost in bankruptcy. The firm can offer $\beta > 0$ and make the manager pay $a = a_S$ for $[NPV_S - NPV_R] + (1 - \alpha)(p_S - p_R)k \geq 0$. Thus, we have proven the next proposition that gives the optimal β under full disclosure.

Proposition 5 *The firm offers* $\beta^{FD} = -\frac{\alpha}{1-\alpha}\frac{NPV_S - NPV_R}{p_S - p_R} > 0$ *only when* $X \in [-(1-\alpha)(p_S - p_R)k, 0)$*, where* $X = NPV_S - NPV_R$*. The manager pays* $a = a_S$ *for all* $X \geq -(1-\alpha)(p_S - p_R)k$ *and selects* a_R *otherwise.*

Under full disclosure, a different kind of inefficiency arises, because the firm prefers S to R even when it is not efficient to select a_S. Thus, this leads to overinvestment in the safe project. This is an unusual feature that never occurred in the prior case when the firm did not select β. Ultimately, the higher bonus counteracts the differential preferences between the firm and the social planner. The firm prefers the safe project to the risky project even though the planner prefers the risky project to the safe project. But the firm can set a solvency bonus optimally to secure its own preference.

The intuition for this is straightforward. Under full disclosure, the manager will select the efficient project without any bonus payment. While this maximizes total surplus (the joint payoff of both the firm and the manager), it is not optimal for the firm. If the risky project has sufficiently higher value than the safe project, then the firm will indeed induce the manager to select the risky project, and this aligns with efficiency. However, if the risky project is insufficiently greater value than the safe project, then there is incentive misalignment. The firm prefers the safe project (because of the negative payoffs in bankruptcy) even though it is efficient for the manager to pick the safe project. In this intermediate region, the firm will pay a positive bonus to the manager to induce him to pick the project that has a higher probability of success (the safe project). Therefore, for risky projects only slightly more valuable than safe projects, the firm overinvests in safety in order to avoid bankruptcy. The firm can contract around the excessive risk-taking of the manager so much that it reduces the underinvestment in safety (overinvestment in risk) and replaces it with an overinvestment in safety (underinvestment in risk).

4.2 No disclosure

Now suppose the firm makes no disclosure to the investor. As before, the investor must form beliefs about the manager's action choice. We can still solve for the optimal solvency bonus that the firm will pick.

Proposition 6 *The firm offers* $\beta^{ND} = -\frac{\alpha}{1-\alpha}\frac{X_S}{p_S - p_R} > 0$ *only when* $X_S \in [-(1-\alpha)(p_S - p_R)k, 0)$ *for* $i = S, R$. *The manager selects* a_S *for all* $X_S \geq -(1-\alpha)(p_S - p_R)k$ *and selects* a_R *otherwise, where* $X_S = NPV_S - NPV_R - p_S(\hat{r_S} - r_S)I - p_R(r_R - \hat{r_S})I$

Without disclosure, the investor must form beliefs about the manager's actions, and we represent those beliefs with r_σ, the interest rate the investor charges when he believes the manager picks a_σ. In the two corner cases, we examine r_S and r_R. This generates two conditions for X_i, depending on whether the investor believes the manager picked the safe project or the risky project (r_S or r_R). The optimal contract that the owner offers will be a function of X_i because it depends on the investor's belief. As before, the firm faces a tension with efficiency for those risky projects that are slightly more valuable than safe projects. For these projects, the firm is worse off with a risky choice even though it is efficient to choose the risky project. The firm will select a positive solvency bonus to counteract the manager's incentive to pick the risky project, thereby inducing overinvestment in the safe project.

Notice that the optimal β varies for different ranges of parameter values under full disclosure and non-disclosure. For the range of parameter values where β is positive under both regimes, β is smaller under the full disclosure regime ($\beta^{FD} < \beta^{ND}$). This should be intuitive because β is a means of motivating the manager to pick the efficient project. Under full disclosure, there is no inefficiency, and therefore, a smaller compensation is necessary. Alternatively, without disclosure, there is a high amount of inefficiency, and therefore, the manager must receive a larger payment in order to reduce this cost to the firm.

4.3 Partial disclosure

Finally, suppose the firm partially discloses the signal to the investor. Now the interest rate the investor offers is a function of both his beliefs as well as a noisy signal of the manager's true action choice.

Proposition 7 *The firm offers* $\beta^{PD} = -\frac{\alpha}{1-\alpha}\frac{X}{p_S - p_R} > 0$ *only when* $X \in [-(1-\alpha)(p_S - p_R)k,\, 0)$. *The managers pays* $a = a_S$ *for all* $X \geq -(1-\alpha)(p_S - p_R)k$ *and selects* a_R *otherwise, where*

$$X = NPV_S + p_S\,(r_S - \rho\tilde{r_S} - (1-\rho)\tilde{r_R})\,I - NPV_R - p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I.$$

Notice that under full disclosure, the manager makes the efficient investment decision. However, the firm has incentives to distort incentives for the manager. Under full disclosure, $X = NPV_S - NPV_R$. Thus, whenever $NPV_S > NPV_R$, the manager invests in the safe project, and the firm does not offer $\beta > 0$. However, when $X \in [-(1-\alpha)(p_S - p_R)k,\, 0)$, the firm offers $\beta > 0$, which leads to overinvestment (in the safe project) by the manager:

Under no disclosure, the manager offers $\beta > 0$ when $X_i \in [-(1-\alpha)(p_S - p_R)k,\, 0)$, where X_S and X_R are defined in the proposition. Suppose X_S holds. A positive β is offered if $X_S > -(1-\alpha)(p_S - p_R)k$. Thus, we have the following kind of inefficiency:

Corollary 1 *There exists a threshold* $k^* \equiv \frac{1}{(1-\alpha)(p_S - p_R)}\Big[p_s(\hat{r_s} - r_s)I + p_R(r_R - \hat{r_s})\Big]$*:*

$$\begin{cases} k > k^* & \Rightarrow \text{under-investment in safe} \\ k = k^* & \Rightarrow \text{efficiency} \\ k < k^* & \Rightarrow \text{over-investment in safe.} \end{cases}$$

Recall that k is the firm's bankruptcy payoff, where higher k reflects worse terminal outcomes for the firm. Taken together, our proposition shows that when the firm's payoffs deteriorate in bankruptcy, this will lead to underinvestment in the safe project. Conversely, when the firm's terminal payoffs are strong and k is low, this will lead to overinvestment in the safe project.

In our model, disclosure facilitates efficiency by improving incentives of the manager to invest in the safe project. The exact same logic works for the firm. The firm faces a loss $(-k)$ when there is bankruptcy. Thus, the cost of bankruptcy is a constant for the firm. So as we move from full disclosure to partial disclosure to no disclosure, the spread between the safe interest rate and the risky interest rate decreases. This makes investment in the safe project less profitable for both the manager and the firm.

Now, whenever the firm prefers the risky project over the safe project, the manager's and the firm's incentives are perfectly aligned. However, there could be a situation where the manager prefers to invest in the risky project, but the firm prefers the safe project. In such situations, as we increase disclosure, the firm's preference tilts towards the safe project, which leads to a positive β offered to the manager to induce investment in the safe project.

Thus, disclosure in such cases leads to a larger set of parameter values for which the firm inefficiently induces the manager to invest in the safe project. However, the firm also has to pay the manager more under the disclosure regime.

5 Conclusion

Disclosure and incentives are two equivalent instruments to resolve the managerial moral hazard problem. In general, more disclosure helps align the manager's action with efficiency. In particular, a lack of disclosure forces the investor to form beliefs about the manager's action. These uncertain beliefs cause him to soften his interest rates, paying a higher interest rate for safe projects and a lower interest rate for risky projects. This shrinking of the spread in interest rates lowers the cost to the manager of selecting the risky project, and the manager responds by increasing risk. Once the firm optimally chooses its contract through a solvency bonus, it can reverse this effect by paying for project success. Indeed, that not only eliminates the overinvestment in risk but generates a second source of inefficiency, overinvestment in safety. Thus, the optimal contract cannot completely eliminate inefficiency in project selection.

Returning to our opening discussion of the financial crisis, how do these results apply to policy lessons? When contracts are not optimally designed, managers are induced to take excessive risk. Under these conditions, disclosure can mitigate this incentive distortion, inducing the manager to reduce this excessive risk-taking. In this sense, disclosure is a powerful instrument to control the agency problem inside the firm. However, when contracts are optimally chosen, firms will structure their bonus to avoid unfavorable bankruptcy. This can reverse the over-investment in risk to generate an over-investment in safety.

6 Appendix

Proof of Proposition 1: The manager selects a_S if and only if

$$NPV_S - NPV_R + p_S(r_S - r_\sigma)I - p_R(r_R - r_\sigma)I \geq 0\,. \tag{10}$$

When $\theta = \theta_i$ the investor observes $\sigma = i$ and the market interest rate is $r_\sigma = r_i$ for $i = R, S$. The payoff from selecting a_S is $(1-\varepsilon)[NPV_S + p_S(r_S - r_{\sigma=S})I] + \varepsilon[NPV_R + p_R(r_R - r_{\sigma=R})I]$ and payoff from selecting a_R is $(1-\varepsilon)[NPV_R + p_R(r_R - r_{\sigma=R})I] + \varepsilon[NPV_S + p_S(r_S - r_{\sigma=S})I]$. Comparing the two expressions, we get the following optimal strategy with disclosure:

$$a = \begin{cases} a_S, & \text{iff} \quad NPV_S \geq NPV_R\,, \\ a_R, & \text{otherwise.} \end{cases} \tag{11}$$

The optimal investment strategy for the manager is to select the a_S project if and only if $NPV_S \geq NPV_R$ and select the a_R project otherwise. ■

Proof of Proposition 2: Assume $p_R \leq p_S$ and $V_{GR} > V_{GS} > I > V_{BS} > V_{BR} > 0$. Zero profit conditions for investors give:

$$(1 + r_R)p_R I + (1 - p_R)V_{BR} = I\,, \tag{12}$$

$$(1 + r_S)p_S I + (1 - p_S)V_{BS} = I\,. \tag{13}$$

Efficiency requires: Invest in i if $NPV_i \geq NPV_j$, for each $j \neq i \in \{R, S\}$, or

$$p_i V_{Gi} + (1 - p_i)VB_i \geq p_j V_{Gj} + (1 - p_j)V_{Bj}\,. \tag{14}$$

Suppose $NPV_S \geq NPV_R$. The manager earns $\alpha\,[V_{Gi} - (1+r_i)I]$ if success and zero otherwise, in expectation, earns

$$EU_i = \alpha p_i \Big(V_{Gi} - (1+r_i)I \Big)\,. \tag{15}$$

Now the manager selects a_S if the expected gain from investing in the safe project is higher than the expected gains from investing in the risky project. That is

$$\alpha\,[(1-\varepsilon)[p_S V_{GS} - p_S(1+r)I] + \varepsilon[p_R V_{GR} - p_R(1+r)I]] \geq$$

$$[\alpha\varepsilon[p_S V_{GS} - p_S(1+r)I] + (1-\varepsilon)[p_R V_{GR} - p_R(1+r)I]\,]\,. \tag{16}$$

Canceling α from both sides we get

$$(1-2\varepsilon)[p_S V_{GS} - p_S(1+r)I] \geq (1-2\varepsilon)[p_R V_{GR} - p_R(1+r)I]\,, \tag{17}$$

$$\text{iff} \quad [p_S V_{GS} - p_S(1+r)I] \geq [p_R V_{GR} - p_R(1+r)I], \tag{18}$$

$$\text{iff} \quad NPV_S - NPV_R + p_S(r_S - r)I - p_R(r_R - r)I \geq 0\,, \tag{19}$$

where (19) is derived by using (12) and (13).

We first want to show that $p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I \geq p_S(\hat{r_S} - r_S)I + p_R(r_R - \hat{r_S})I$. This directly follows from $p_S \geq p_R$.

In equilibrium, actions must match beliefs. Thus, the equilibrium interest rate must be $\hat{r_S}$ whenever the manager plays a_S and $\hat{r_R}$ whenever the manager plays a_R. Consider parameter values such that $NPV_S - NPV_R = p_S(r - r_S)I + p_R(r_R - r)I$ and condition (6) holds.

Clearly, $NPV_S - NPV_R > p_S(\hat{r_S} - r_S)I + p_R(r_R - \hat{r_S})I$, which makes it optimal for the manager to invest in the safe project along with the market interest being $\hat{r_S}$.

Also, $NPV_S - NPV_R < p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I$ makes it optimal for the manager to play a_R in addition to the market interest being $\hat{r_R}$.

Step 1: Suppose the market interest rate is $\hat{r_R}$. The manager selects a_S if

$$NPV_S - NPV_R + p_S(r_S - \hat{r_R})I - p_R(r_R - \hat{r_R})I \geq 0, \tag{20}$$

$$\text{iff} \quad NPV_S \geq NPV_R + p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I. \tag{21}$$

Now suppose (21) holds. We want to see if the inequality still holds with $r = \hat{r_S}$. Since $p_S \geq p_R$, $NPV_R + p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I \geq NPV_R + p_S(\hat{r_S} - r_S)I + p_R(r_R - \hat{r_S})I$. So the above equation holds with $r = \hat{r_S}$.

Step 2: Suppose $NPV_S < NPV_R + p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I$. With $r = \hat{r_R}$, the manager selects a_R. We know that with $r = \hat{r_S}$ the manager selects a_S if $NPV_S \geq NPV_R + p_S(\hat{r_S} - r_S)I + p_R(r_R - \hat{r_S})I$. So the manager selects a_R, with $r = \hat{r_R}$, if

$$NPV_S < NPV_R + p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I. \tag{22}$$

Suppose that $NPV_S < NPV_R + p_S(\hat{r_S} - r_S)I + p_R(r_R - \hat{r_S})I$. We want to check if this implies that (22) holds. Now, we can show that

$$NPV_R + p_S(\hat{r_S} - r_S)I + p_R(r_R - \hat{r_S})I \leq NPV_R + p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I. \tag{23}$$

which follows from $p_S \geq p_R$, so (22) holds.

■

Lemma 1 $(\tilde{r_R} - \bar{r_R}) = (\tilde{r_S} - \bar{r_S}) > 0$

Proof of Lemma 1: We can write $(\tilde{r_S} - \bar{r_S})$ as

$$(\tilde{r_S} - \bar{r_S}) = [ar_R + (1-a)r_S] - [br_R + (1-b)r_S], \tag{24}$$

where

$$a = Pr(\theta_R|S, \mu = 1) = \frac{(1-\rho)(1-\varepsilon)}{(1-\rho)(1-\varepsilon) + \rho\varepsilon}, \tag{25}$$

and

$$b = Pr(\theta_R|S, \mu = 0) = \frac{(1-\rho)\varepsilon}{(1-\rho)\varepsilon + \rho(1-\varepsilon)}. \tag{26}$$

Similarly, we can write $(\tilde{r_R} - \bar{r_R})$ as,

$$(\tilde{r_R} - \bar{r_R}) = [cr_R + (1-c)r_S] + [dr_R + (1-d)r_S], \tag{27}$$

where $c = Pr(\theta_R|R, \mu = 1) = \frac{\rho(1-\varepsilon)}{\rho(1-\varepsilon)+(1-\rho)\varepsilon} = (1-b)$ and $d = Pr(\theta_R|R, \mu = 0) = \frac{\rho\varepsilon}{\rho\varepsilon+(1-\rho)(1-\varepsilon)} = 1-a$. Therefore,

$$(\tilde{r_R} - \bar{r_R}) - (\tilde{r_S} - \bar{r_S}) = [(a+d) - (b+c)](r_R + r_S) = 0. \tag{28}$$

■

Proof of Proposition 3: The probability table is

$\sigma\backslash\theta$	θ_R	θ_S
R	p_{RR}	p_{RS}
S	p_{SR}	p_{SS}

By Bayes Rule,

$$Pr(a_S|S) = \frac{p_{SS}[\mu\varepsilon + (1-\mu)(1-\varepsilon)]}{p_{SS}[\mu\varepsilon + (1-\mu)(1-\varepsilon)] + p_{SR}\cdot[\mu(1-\varepsilon)+(1-\mu)\varepsilon]}, \tag{29}$$

$$Pr(a_R|R) = \frac{p_{RR}\cdot[\mu(1-\varepsilon)+(1-\mu)\varepsilon]}{p_{RR}\cdot[\mu(1-\varepsilon)+(1-\mu)\varepsilon] + p_{RS}[(1-\mu)(1-\varepsilon)+\mu\varepsilon]}, \tag{30}$$

$$Pr(a_S|R) = \frac{p_{RS}[(1-\mu)(1-\varepsilon)+\mu\varepsilon]}{p_{RR}\cdot[\mu(1-\varepsilon)+(1-\mu)\varepsilon] + p_{RS}[(1-\mu)(1-\varepsilon)+\mu\varepsilon]}, \tag{31}$$

$$Pr(a_R|S) = \frac{p_{SR}\cdot[\mu(1-\varepsilon)+(1-\mu)\varepsilon]}{p_{SS}[\mu\varepsilon + (1-\mu)(1-\varepsilon)] + p_{SR}\cdot[\mu(1-\varepsilon)+(1-\mu)\varepsilon]}. \tag{32}$$

The manager selects a_S if the expected gain from investing in the safe project is higher than the expected gains from investing in the risky project. That is,

$$(1-\varepsilon)\left[NPV_S + p_S\Big(r_S - E(r_\sigma,\theta_S)\Big)I\right] + \varepsilon\left[NPV_R + p_R\Big(r_R - E(r_\sigma,\theta_R)\Big)I\right] \geq$$

$$\varepsilon\left[NPV_S + p_S\Big(r_S - E(r_\sigma,\theta_S)\Big)I\right] + (1-\varepsilon)\left[NPV_R + p_R\Big(r_R - E(r_\sigma,\theta_R)\Big)I\right], \tag{33}$$

if and only if

$$NPV_S + p_S\Big(r_S - E(r_\sigma,\theta_S)\Big)I \geq NPV_R + p_R\Big(r_R - E(r_\sigma,\theta_R)\Big)I, \tag{34}$$

where

$$E(r_\sigma,\theta_i) = Pr(S|\theta_i)\Big(Pr(\theta_S|S)r_S + Pr(\theta_R|S)r_R\Big)$$
$$+Pr(R|\theta_i)\Big(Pr(\theta_S|R)r_S + Pr(\theta_R|R)r_R\Big), \tag{35}$$

is the interest rate the market charges in expectation when the true state is $\theta_i = \theta_R, \theta_S$. Clearly, as p_{RR}, $p_{SS} \to 1$, this approximates the efficiency condition.

In equilibrium, beliefs must coincide with actions. Thus, the following condition gives the threshold above, which investing in the safe asset is a dominant strategy for the manager.

Suppose $\hat{\mu} = 1$. The manager selects a_S if

$$NPV_S - NPV_R \geq p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I - p_S\left(r_S - \rho\tilde{r_S} - (1-\rho)\tilde{r_R}\right)I. \quad (36)$$

Similarly, the following condition depicts the threshold below which investing in the risky project is a dominant strategy for the manager.

When $\hat{\mu} = 0$, the manager selects a_S if

$$NPV_S - NPV_R \geq p_R\Big(r_R - (1-\rho)\bar{r_S} - \rho\bar{r_R}\Big)I - p_S\Big(r_S - \rho\bar{r_S} - (1-\rho)\bar{r_R}\Big)I. \quad (37)$$

First, we want to show that, RHS (37) $\leq$ RHS (36). We use the following Lemma to get to this result.

Comparing (36) and (37) and using Lemma 1,

$$\begin{aligned}
& p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I - p_S\left(r_S - \rho\tilde{r_S} - (1-\rho)\tilde{r_R}\right)I \\
- \;& p_R\Big(r_R - (1-\rho)\bar{r_S} - \rho\bar{r_R}\Big)I + p_S\Big(r_S - \rho\bar{r_S} - (1-\rho)\bar{r_R}\Big)I \\
= \;& \Big[p_R((1-\rho)\bar{r_S} + \rho\bar{r_R} - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}) - p_S\Big(\rho\bar{r_S} + (1-\rho)\bar{r_R} - \rho\tilde{r_S} - (1-\rho)\tilde{r_R}\Big)\Big]I \\
= \;& [p_R\{(1-\rho)(\bar{r_S} - \tilde{r_S}) + \rho(\bar{r_R} - \tilde{r_R})\} - p_S\{\rho(\bar{r_S} - \tilde{r_S}) + (1-\rho)(\bar{r_R} - \tilde{r_R}\}]I \quad (38) \\
. & \quad (39)
\end{aligned}$$

Using Lemma 1, the above expression reduces to

$$(p_R - p_S)(\bar{r_S} - \tilde{r_S})I < 0.$$

Case 1: Condition (9) holds. Consider parameter values such that $NPV_S - NPV_R = p_S(E(r_\sigma, \theta_S) - r_S)I + p_R(r_R - E(r_\sigma, \theta_R))I$ and condition (9) holds.

Clearly, $NPV_S - NPV_R \geq p_R\Big(r_R - (1-\rho)\bar{r_S} - \rho\bar{r_R}\Big)I - p_S\Big(r_S - \rho\bar{r_S} - (1-\rho)\bar{r_R}\Big)I$, which makes it optimal for the manager to invest in the safe project along with the market interest being $\bar{r_S}$ when $\sigma = S$ and $\bar{r_R}$ when $\sigma = R$.

Also, $NPV_S - NPV_R < p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I - p_S\left(r_S - \rho\tilde{r_S} - (1-\rho)\tilde{r_R}\right)I$ makes it optimal for the manager to play a_R along with the market interest being $\tilde{r_S}$ when $\sigma = S$ and $\tilde{r_R}$ when $\sigma = R$.

Thus, for each combination of parameter values satisfying (9), we find a threshold above (below) which the manager invests in the safe(risky) project.

Case 2: Condition (9) does not hold. Suppose $NPV_S - NPV_R > p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I - p_S\left(r_S - \rho\tilde{r_S} - (1-\rho)\tilde{r_R}\right)I$. Clearly, investing in the safe project is optimal for the manager for this range of parameter values. Now, for this range we also have $NPV_S - NPV_R > p_R\Big(r_R - (1-\rho)\bar{r_S} - \rho\bar{r_R}\Big)I - p_S\Big(r_S - \rho\bar{r_S} - (1-\rho)\bar{r_R}\Big)I$ consistent with the beleif $\hat{\mu} = 0$ and market interest rates $\bar{r_S}$ and $\bar{r_R}$. Thus, in equilibrium, the manager invests in the safe project whenever $NPV_S - NPV_R > p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I - p_S\left(r_S - \rho\tilde{r_S} - (1-\rho)\tilde{r_R}\right)I$.

When $NPV_S - NPV_R < p_R\Big(r_R - (1-\rho)\bar{r_S} - \rho\bar{r_R}\Big)I - p_S\Big(r_S - \rho\bar{r_S} - (1-\rho)\bar{r_R}\Big)I$, the manager's optimal action is to invest in the risky asset. Now, for this range of parameter vales, we also have $NPV_S - NPV_R < p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I - p_S\left(r_S - \rho\tilde{r_S} - (1-\rho)\tilde{r_R}\right)I$, which is consistent with the belief $\hat{\mu} = 1$ and market interest rates $\tilde{r_S}$ and $\tilde{r_R}$. Therefore, in equilibrium, the manager invests in the risky project when $NPV_S - NPV_R < p_R\Big(r_R - (1-\rho)\bar{r_S} - \rho\bar{r_R}\Big)I - p_S\Big(r_S - \rho\bar{r_S} - (1-\rho)\bar{r_R}\Big)I$.

■

Proof of Propositon 4: No Disclosure. When there is no disclosure, the inefficiency occurs for $NPV_S - NPV_R \geq p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I$ and for $NPV_S - NPV_R \geq p_S(\hat{r_S} - r_S)I + p_R(r_R - \hat{r_S})I$, where $\hat{r}_R$ and $\hat{r_S}$ is given by (5) and (4).

Since $p_S > p_R$,

$$p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I > p_S(\hat{r_S} - r_S)I + p_R(r_R - \hat{r_S})I. \tag{40}$$

Thus, the maximum range for which we have inefficiency under no disclosure is given by

$$NPV_S - NPV_R \in (0,\, p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I). \tag{41}$$

The minimum range for which we have inefficiency under no disclosure is given by

$$NPV_S - NPV_R \in (0,\, p_S(\hat{r_S} - r_S)I + p_R(r_R - \hat{r_S})I). \tag{42}$$

Define $\overline{\beta} = \frac{1-\alpha}{\alpha}\beta$. The manager selects a_S if

$$\begin{aligned}(1-\varepsilon)[p_S V_{GS} - p_S(1+r)I + p_S\overline{\beta}] + \varepsilon[p_R V_{GR} - p_R(1+r)I + p_R\overline{\beta}] \\ \geq \varepsilon[p_S V_{GS} - p_S(1+r)I + p_S\overline{\beta}] + (1-\varepsilon)[p_R V_{GR} - p_R(1+r)I + p_R\overline{\beta}],\end{aligned} \tag{43}$$

$$\text{iff} \quad (1-2\varepsilon)[p_S V_{GS} - p_S(1+r)I + p_S\overline{\beta}] \geq (1-2\varepsilon)[p_R V_{GR} - p_R(1+r)I + p_R\overline{\beta}]\,, \quad (44)$$

$$\text{iff} \quad [p_S V_{GS} - p_S(1+r)I] + \overline{\beta}(p_S - p_R) \geq [p_R V_{GR} - p_R(1+r)I], \quad (45)$$

$$\text{iff} \quad NPV_S - NPV_R + \overline{\beta}(p_S - p_R) + p_S(r_S - r)I - p_R(r_R - r)I \geq 0\,. \quad (46)$$

For $NPV_S - NPV_R \in (0,\, p_S(\hat{r_R} - r_S)I + p_R(r_R - \hat{r_R})I]$, we require

$$\overline{\beta}(p_S - p_R) + p_S(r_S - r)I - p_R(r_R - r)I = 0. \quad (47)$$

That is

$$\beta^* = \frac{p_S(\hat{r_S} - r_S)I + p_R(r_R - \hat{r_S})I}{p_S - p_R}\frac{\alpha}{1-\alpha}. \quad (48)$$

This leads to efficient risk taking by the manager.

Now suppose the maximum bonus permissible is less than the efficient β.

Noisy Signals. The maximum range for which we have inefficiency under partial disclosure is given by (follows from Proposition 3):

$$NPV_S + p_S\left(r_S - \rho\tilde{r_S} - (1-\rho)\tilde{r_R}\right)I \geq NPV_R + p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I. \quad (49)$$

To achieve efficiency for this range of parameter values we require

$$NPV_S - NPV_R + \overline{\beta}(p_S - p_R) - p_S\left(\rho\tilde{r_S} + (1-\rho)\tilde{r_R} - r_S\right)I - p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I \geq 0. \quad (50)$$

So the efficient β is given by

$$\beta^{**} = \frac{p_S\left(\rho\tilde{r_S} + (1-\rho)\tilde{r_R} - r_S\right)I + NPV_R + p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I}{(p_S - p_R)}\frac{\alpha}{1-\alpha}. \quad (51)$$

■

Proof of Proposition 6: *Consider the manager's problem.*

The manager selects a_S if

$$(1-\varepsilon)\left\{\alpha[p_S V_{GS} - p_S(1+r)I - p_S\beta] + p_S\beta\right\} + \varepsilon\left\{\alpha[p_R V_{GR} - p_R(1+r)I - p_R\beta] + p_R\beta\right\}$$

$$\geq \varepsilon\left\{\alpha[p_S V_{GS} - p_S(1+r)I - p_S\beta] + p_S\beta\right\} + (1-\varepsilon)\left\{\alpha[p_R V_{GR} - p_R(1+r)I - p_R\beta] + p_R\beta\right\}$$

$$\Leftrightarrow (1-2\varepsilon)\left\{\alpha[p_S V_{GS} - p_S(1+r)I] + (1-\alpha)p_S\beta\right\}$$

$$\geq (1-2\varepsilon)\left\{\alpha[p_R V_{GR} - p_R(1+r)I - p_R\beta] + (1-\alpha)p_R\beta\right\}$$

$$\Leftrightarrow \left\{\alpha[p_S V_{GS} - p_S(1+r)I] + (1-\alpha)p_S\beta\right\}$$
$$\geq \left\{\alpha[p_R V_{GR} - p_R(1+r)I - p_R\beta] + (1-\alpha)p_R\beta\right\}$$

$$\alpha\left[p_S V_{GS} - p_S(1+r)I - p_R V_{GR} + p_R(1+r)I\right] + (1-\alpha)\beta(p_S - p_R) \geq 0. \quad (52)$$

Define $X = p_S V_{GS} - p_S(1+r)I - p_R V_{GR} + p_R(1+r)I$ so that (52) reduces to

$$\alpha X + (1-\alpha)\beta(p_S - p_R) \geq 0. \quad (53)$$

As we have seen in earlier sections as well, when $\beta = 0$, the manager invests ineffeciently when NPV_S is not sufficiently higher than NPV_R. However, the manager invests efficiently when $NPV_S < NPV_R$.

Consider the firm's problem.

Note that $X = NPV_S + p_S(r_S - r)I - NPV_R - p_R(r_R - r)I$. Now, the firm's problem is to select a_S if

$$(1-\varepsilon)\left\{(1-\alpha)[p_S V_{GS} - p_S(1+r)I - p_S\beta] - (1-p_S)k\right\} +$$
$$\varepsilon\left\{(1-\alpha)[p_R V_{GR} - p_R(1+r)I - p_R\beta] - (1-p_R)k\right\}$$
$$\geq \varepsilon\left\{(1-\alpha)[p_S V_{GS} - p_S(1+r)I - p_S\beta] - (1-p_S)k\right\} +$$
$$(1-\varepsilon)\left\{(1-\alpha)[p_R V_{GR} - p_R(1+r)I - p_R\beta] - (1-p_R)k\right\}.$$

This becomes

$$\left\{(1-\alpha)[p_S V_{GS} - p_S(1+r)I - p_S\beta] - (1-p_S)k\right\}$$
$$\geq \left\{(1-\alpha)[p_R V_{GR} - p_R(1+r)I - p_R\beta] - (1-p_R)k\right\}.$$

Simplifying,

$$(1-\alpha)\left[p_S V_{GS} - p_S(1+r)I - p_R V_{GR} + p_R(1+r)I\right] + (1-\alpha)(p_S - p_R)[k-\beta] \geq 0, \quad (54)$$

$$\Leftrightarrow (1-\alpha)X + (1-\alpha)(p_S - p_R)[k-\beta] \geq 0. \quad (55)$$

When there is no disclosure, the market charges a fixed interest rate. Under no disclosure, if $\beta = 0$, the manager selects a_S if $\alpha X \geq 0$. However, a_S is optimal for the firm when $(1-\alpha)X + (1-\alpha)(p_S - p_R)[k - \beta] \geq 0$. Thus, in equilibrium for the owner to offer a positive β and the manager to invest in the safe project the following three conditions must hold:

1. $\alpha X + (1-\alpha)\beta(p_S - p_R) \geq 0$
2. $(1-\alpha)X + (1-\alpha)(p_S - p_R)[k - \beta] \geq 0$ and
3. $r = \hat{r_S}$

Notice that $X \geq 0$ implies that (55) is satisfied. Therefore, whenever the manager has incentives to invest in the safe project without bonus, a_S is also the optimal action for the owner.

Now let's turn our attention to our case of interest, that is, $X < 0$ and $(1-\alpha)X + (1-\alpha)(p_S - p_R)[k - \beta] \geq 0$. The manager has no incentives to invest in the safe project without a positive bonus, but the owner prefers the action a_S. Now,

$$X = X_S = NPV_S - NPV_R - p_S(\hat{r}_S - r_s)I - p_R(r_R - \hat{r}_S)I. \tag{56}$$

Suppose $X_S < 0$, or $NPV_S - NPV_R < p_S(\hat{r_S} - r_S)I + p_R(r_R - \hat{r_S})I$.

The minimum β required to make the manager play a_S is given by

$$(1-\alpha)\beta(p_S - p_R) = -\alpha X_S\,. \tag{57}$$

The maximum willingness to pay by the firm is given by the following condition:

$$(1-\alpha)\beta(p_S - p_R) = (1-\alpha)X_S + (1-\alpha)(p_S - p_R)k\,. \tag{58}$$

Thus, as long as $(1-\alpha)X_S + (1-\alpha)(p_S - p_R)k \geq -\alpha X$, the firm will offer to transfer β. The above condition is summarized by

$$X_S + (1-\alpha)(p_S - p_R)k \geq 0\,. \tag{59}$$

From (57) we obtain

$$\beta^{ND} = -\frac{\alpha}{1-\alpha}\frac{X_S}{p_S - p_R} > 0\,.$$

■

Proof of Proposition 7: From the proof of Proposition 3, we know that the manager selects a_S if

$$NPV_S + p_S\Big(r_S - E(r_\sigma, \theta_S)\Big)I \geq NPV_R + p_R\Big(r_R - E(r_\sigma, \theta_R)\Big)I\,, \tag{60}$$

where

$$E(r_\sigma, \theta_i) = Pr(S|\theta_i)\Big[Pr(\theta_S|S)r_S + Pr(\theta_R|S)r_R\Big] + Pr(R|\theta_i)\Big[Pr(\theta_S|R)r_S + Pr(\theta_R|R)r_R\Big] \tag{61}$$

is the interest rate the market charges in expectation when the true state is i.

In equilibrium, the manager selects a_S if $NPV_S + p_S\,(r_S - \rho\tilde{r_S} - (1-\rho)\tilde{r_R})\,I \geq NPV_R + p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I$ and selects a_R otherwise. Therefore, the firm does not need to give a solvency contingent bonus when $X = NPV_S + p_S\,(r_S - \rho\tilde{r_S} - (1-\rho)\tilde{r_R})\,I - NPV_R - p_R\Big(r_R - (1-\rho)\tilde{r_S} - \rho\tilde{r_R}\Big)I \geq 0$.

Moreover, when $X \geq 0$, we have $(1-\alpha)X + (1-\alpha)(p_S - p_R)k \geq 0$ for all $k \geq 0$, that is, the firm also prefers selecting a_S.

When $X < 0$, the manager selects a_R, but the firm prefers S if $(1-\alpha)X + (1-\alpha)(p_S - p_R)k \geq 0$. However, the maximum willingness to pay by the firm exceeds the minimum bonus required to make the manager pay a_S only if $X \in [-(1-\alpha)(p_S - p_R)k,\, 0)$. This follows from (59).

Thus, in this range, the bonus that will be offered is given by (57), that is,

$$\beta = -\frac{\alpha}{1-\alpha}\frac{X}{p_S - p_R}.$$

■

References