Competition: Boon or Bane for Reputation Building Behavior*

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Abstract

This paper investigates whether competition aids or hinders reputation building behavior in experience goods markets. We examine a market where long lived firms face a short term incentive to put low effort. There are two types of firms, "good" firms to whom high effort is costless and "opportunistic" firms who have to pay a small cost for high effort. We characterize the equilibrium strategies of a monopolist and a duopolist for a two period model. Contrary to the prevalent idea that competition improves reputation building behavior we find that competition may hinder reputation building behavior by shrinking expected future payoffs. Horner (2002) talks about a perfectly competitive market and emphasizes the importance of outside options generated through competition. Our model on the other hand compares a duopoly model with a monopoly model in an environment of price competition. We provide an analytical framework and explicitly derive conditions for which competition aids reputation-building behavior and for which it hiders. We also examine the case where a planner can observe the hired firm's type and can dictate the chosen firm's actions. We show under such circumstances the duopolist's choice of effort coincides or falls below the effort level the planner prescribes.

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1 Introduction

On one hand, competition enhances reputation-building behavior, because dissatisfied clients have the option of switching firms. The fear of losing customers to rivals forces firms to keep their reputations. This view is formulated by Klein and Leffler (1981), and later Horner (2002) and Vial (2010).

However, there is another effect that works in an opposite direction. Competition reduces expected future payoff which precludes a firm from putting high effort. Kranton (2003) points out the second effect. By means of numerical examples, Bar-Issac (2005) shows that competition has non-monotone effects to reputational commitments for quality. [1]

In this paper, we provide an analytical framework and explicitly derive conditions for which competition aids reputation-building behavior and for which it hinders. This is a simple two-period model. The economy consists of long-lived seller(s) and short-lived buyers. In each period, every seller who is matched with a buyer produces an indivisible good, which can be either of high or of low quality. There are two types of sellers: good or opportunistic. A good seller always produces high-quality goods without any cost. An opportunistic seller incurs a positive cost to produce high-quality goods and may or may not mimic a good seller.

We compare two cases, namely, *monopoly* and *duopoly*. In case of monopoly, there is only one long-lived seller. In the first period, the seller has a buyer and he decides on the quality (if he is opportunistic). In the second period, the seller posts price. A potential buyer shows up with probability half. She observes the quality of the product in the first period as well as the price, and then decides whether to purchase from the seller.

In case of duopoly, there are two ex ante identical long-lived sellers who compete for the buyers. In the first period, a seller has a buyer while the other doesn't. A seller who has a buyer decides on the quality of output. In the second period, both sellers post prices. In this case, sellers engage in Bertrand competition. Then a buyer shows up. Hence, the ratio of sellers and buyers are the same regardless of monopoly or duopoly. She observes the quality

[1] Bar-Issac and Tadelis (2008) provide a convenient survey.

of the product that a seller made in the first period as well as the prices. Then she decides whether to purchase from a seller, and if so, from whom.

The main finding of the paper is as follows. First, if the ex ante probability of seller(s) being good is more than half, then monopoly outperforms duopoly in terms of effort level. Second, if the ex ante probability of seller(s) being good is less than half, then the effort level depends on the difference of utilities of consuming high and low quality. If the difference is sufficiently high, the monopoly effort level is higher than the duopoly effort level, while if it is low then the opposite is true.

In addition we also show that the effort choice of a duopolist either coincides or falls below the effort choice prescribed by a benevolent social planner with minimum control.

2 The Model

Environment: Consider a market in which firms and consumers repeatedly trade. Time is discrete and there are only two periods indexed $t = 1, 2$. Firms are long lived while a new consumer arrives in every period. The consumer may trade with only one firm in a given period. In this event the consumer pays upfront and enjoys a product whose quality depends on the unobserved effort level exerted by the chosen firm. The product can be of high (H) quality or of low (L) quality yielding utility levels v_h and v_l respectively. $v_h > v_l \geq 0$, where the utility from not consuming the good is normalized to be zero. Let $q \in \{h, l\}$ denote the quality of output. Firms are of two types, good or opportunistic, which is private information. A firm is good with probability $\alpha > 0$ and firms' types are drawn independently. Good firms produce high quality goods at no cost while opportunistic firms choose quality. For opportunistic firms cost of producing high quality output is given by $c > 0$ while the cost of producing low quality output is normalized to zero. We assume that $\triangle v = v_h - v_l > c$, that is, it is economically efficient for the opportunistic firm to produce the high quality output.

Timeline: The sequence of events is as follows.

In period 1, the consumer decides from whom to buy and firms decide on the quality of the product.

In period 2, the new consumer observes the quality of the good produced in the last period and updates her belief about the firm's type. Firms post prices (in case of duopoly there is Bertrand competition). The consumer then decides from whom to buy.

Equilibrium concept: In equilibrium,

1. Firms choose effort level in the first period and prices in the second period to maximize their payoff.
2. The consumer behaves optimally in the second period given her belief.
3. Beliefs are updated using Bays rule.
4. Beliefs coincide with actions.

2.1 Monopoly

In the monopoly model there is one long lived firm and a new consumer in each period. In period 1, the consumer decides whether to buy from the monopolist and the monopolist decides on the quality of the product if the consumer is interested in trade. In period 2, a new consumer arrives who observes the quality of the good produced in the last period and updates her belief about the firm's type. The monopolist posts a price. The consumer then decides whether to buy from the monopolist.

Strategies: A stationary strategy for the firm is a pair (P, x), i.e. , a price and a choice of effort. Formally, $x : [0, 1] \to [0, 1]$, where $x(\alpha)$ is the probability that the opportunistic firm exerts effort in period 1 and $P : [0, 1] \to \mathbb{R}$ gives the fee the firm posts in period 2.

A consumer's strategy is whether to buy from a firm given her belief about the firm's type in the second period and the price the firm posts in the second period.

Belief update: Let $\phi(\alpha, q)$ be the belief update function where, $\phi(\alpha, q)$ gives the probability that the firm is good given the prior α and the quality of output $q \in \{h, l\}$.

Clearly, $\phi(\alpha, l) = 0$. This is because the good firm produces a high quality output at no cost and the good firm is assumed to be non-strategic, that is, the good firm only produces a high quality output. Therefore, whenever the consumer observes a low quality output she concludes that the producer of the good is the opportunistic type. When $q = h$, the belief update function obtained using Bays rule is as follows

$$\phi(\alpha, H) = \frac{\alpha}{\alpha + (1 - \alpha)x}$$

where, α is the prior probability that the firm is good and x is the probability that the opportunistic firm exerts effort.

Analysis: Let us first consider the price posting behavior of the firm in period 2. In equilibrium, the firm posts a price that makes the consumer indifferent between purchasing the good and not purchasing it. The firm can do better if the price is such that the consumer strictly prefers trade to autarky. Hence the price must equal the expected utility the consumer obtains from the transaction and the price is given by,

$$P = \phi(\alpha, q)v_h + (1 - \phi(\alpha, q))v_l$$

Given the the belief update rule and the price posting behavior described above, the firm's gains from putting effort and producing a high quality good in the first period is given by $\frac{\alpha}{\alpha+(1-\alpha)x}v_h + \left(1 - \frac{\alpha}{\alpha+(1-\alpha)x}\right)v_l - c$. Instead if the the firm does not put effort in period 1 and produces a low quality output, the consumer becomes sure that the firm is an opportunistic type. In this event the only price the firm can post in period 2 is v_l. The firm will be indifferent between exerting effort and not exerting effort if the following condition holds

$$\frac{\alpha}{\alpha + (1-\alpha)x} v_h + \left(1 - \frac{\alpha}{\alpha + (1-\alpha)x}\right) v_l - c = v_l \tag{1}$$

Solving for x from the above equation we get,

$$x = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\triangle v - c}{c}\right) \tag{2}$$

Equilibrium Conditions: In equilibrium the monopolist posts a price in period 2 which depends on the prior probability that the firm is good, the equilibrium choice of effort by the monopolist in period 1 and the observed quality of the output in period 1. Thus the equilibrium pricing rule is given by

$$P^M = \begin{cases} v_l & if\ q = l \\ \frac{\alpha}{\alpha+(1-\alpha)x^M} v_h + \left(1 - \frac{\alpha}{\alpha+(1-\alpha)x^M}\right) v_l & if\ q = h \end{cases} \tag{3}$$

In equilibrium, the choice of effort of the opportunistic firm in period 1 is given by

$$x^M = \begin{cases} \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\triangle v - c}{c}\right) & if\ \alpha \triangle v < c \\ 1 & if\ \alpha \triangle v \geq c \end{cases} \tag{4}$$

The equilibrium condition specified above implies that the monopolist exerts effort with positive probability for all possible parameter values. The monopolist exerts effort for sure beyond the threshold reputation $c/\triangle v$.

Proposition 1: *x^M is increasing in α and $\triangle v$ while decreasing in c.*

Proof:

Partially differentiating x^M with respect to α, $\triangle v$ and c we get,

$$\frac{\partial x^M}{\partial \alpha} = \left(\frac{\triangle v - c}{c}\right)\frac{1}{(1-\alpha)^2} > 0$$

$$\frac{\partial x^M}{\partial \triangle v} = \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{c} > 0$$

$$\frac{\partial x^M}{\partial c} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{-\triangle v}{c^2}\right) < 0 \qquad \blacksquare$$

The monopolist's choice of effort is increasing in reputation, which means, as the consumer becomes more convinced that the monopolist is of the good type the monopolist can reap the gains from reputation better. This is because a higher reputation is associated with a higher expected quality of the good and hence a higher payment for the monopolist. By not putting effort the opportunistic firm produces a bad quality output and that pushes reputation down to zero. In this event the only price the monopolist can charge in period 2 is v_l. Thus reputation incentives increases as reputation of the firm increases.

Reputation incentives are also increasing in $\triangle v$ that is the increment in quality resulting from high effort as opposed to low effort. For a better understanding of the result, let us fix v_l and vary v_h. Note that in equation (3), for any given level of reputation $\alpha > 0$ the price the monopolist can post in period 2, is increasing in v_h. On the other hand the gains from not putting effort is fixed at v_l. Thus as the difference between the quality of the output increases reputation incentives become stronger.

The third result of Proposition 1 is straight forward. A higher cost of effort is associated with a lower choice of effort. Therefore, as c increases the threshold beyond which the monopolist puts effort with probability 1 increases too. The range of reputation where the monopolist exerts effort with positive probability is now characterized by lower level of effort.

2.2 Duopoly

In this section we discuss a duopoly model with two firms and one consumer. This model captures reputation building behaviors of firms when they compete for clients through price posting. Since this model assumes that there is only one consumer in each period, the results we obtain in this section is also affected by the reduction in market size. In the monopoly model the market size for the firm was given by one consumer and the consumer-seller ratio was 1. In the duopoly model the ratio reduces to 1/2. In this section we analyze the model with consumer-seller ratio 1/2. In the next section we analyze a model that isolates the effect of competition from the market size effect.

There are two long lived firms and a new consumer in each period. Consumers trade only with one firm in each period. In period 1, the consumer chooses a firm and the chosen firm decides on the quality of the product. In period 2, a new consumer arrives who observes the quality of the good produced in the last period and updates her belief about the type of the firm who was hired in period 1. The firms post prices. the consumer then decides from whom to buy.

Strategies: A stationary strategy for the firm is a pair (P, x), i.e. , a price and a choice of effort. Formally, $x : [0, 1] \to [0, 1]$, where $x(\alpha)$ is the probability that the opportunistic firm exerts effort in period 1 and $P : [0, 1] \to \mathbb{R}$ gives the price the firm posts in period 2.

A consumer's strategy specifies from which firm to buy, given her beliefs about the firms' types in the second period and the prices they post in the second period.

Consumer's belief and behavior: The belief update rule is same as described in the monopoly model. $\phi(\alpha, q)$ is the belief update function where, $\phi(\alpha, q)$ gives the probability that the firm is good given the prior α and the quality of output $q \in \{h, l\}$.

Clearly, $\phi(\alpha, l) = 0$. This is because a good firm produces a high quality output at no cost and a good firm is also non-strategic. Therefore, whenever the consumer observes a low quality output she concludes that the producer of the good is the opportunistic type. When $q = h$, the belief update function obtained using Bays rule is as follows

$$\phi(\alpha, H) = \frac{\alpha}{\alpha + (1 - \alpha)x}$$

where, α is the prior probability that the firm is good and x is the probability that the opportunistic firm exerts effort.

Given the belief update rule, the consumer in period 2 purchases only from a firm that gives her higher net expected utility. Net expected utility of a consumer is defined as the difference between the expected utility of the consumer from a transaction and the price the consumer has to pay for the good. In case of a tie, we assume that the consumer purchases from the firm whose product yields higher expected utility. This assumption implies that reputation is valuable to a firm. Even when a rival firm cuts price in order to match the expected net utility of a firm, the consumer trades only with the firm that has a higher reputation.

Analysis: Let us first consider the price posting behavior of the firms in period 2. In equilibrium, the firm posts a price that makes the consumer indifferent between purchasing from him and purchasing from his rival. Therefore, the price a firm posts depends on whether the consumer in period 1 purchased from him, the observed quality of the good produced in period 1 and the current reputation of the firm. Consider the following price posting behavior.

In the second period, the firm who did not have trade in the first period posts price P' such that,

$$P' = \begin{cases} 0 & if\ q = h \\ \alpha v_h + (1 - \alpha)v_l - v_l & if\ q = l \end{cases} \tag{5}$$

The firm who had trade in the first period posts price P such that,

$$P = \begin{cases} (\phi(\alpha, h) - \alpha)\triangle v & if\ q = h \\ 0 & if\ q = l \end{cases} \tag{6}$$

Suppose both firms have same reputation α in the first period. Suppose one firm is chosen by the consumer in the first period. Our model does not assume price posting in the first period in order to set aside the issue of price signaling. The focus of our analysis is to capture the effect of competition on reputation building behavior. For this reason we assume that a firm is chosen in the first period by the consumer for reasons outside the arena of this model. However the firms are strategic when it comes to posting prices in the second period and the consumer in the second period chooses a firm whose product yields a higher net expected utility for the consumer. In case of a tie the consumer trades with the firm that yields a higher expected utility.

Given this selection behavior of the consumer, the above price posting behavior is optimal for the firms. A firm who did not have trade in the first period, must take into account the quality of the good his rival produced in the first period. If the quality of the good was observed to be high in the first period, the rival firm who had trade in the first period moves to a higher level of reputation $\phi(\alpha, h)$. A higher level of reputation is associated with a higher expected utility $\phi(\alpha, h)v_h + (1 - \phi(\alpha, h))v_l$. For any $P' \in (0, \alpha v_h + (1 - \alpha)v_l)$ the rival firm whose reputation is $\phi(\alpha, h)$ can post $P = \phi(\alpha, h)v_h + (1 - \phi(\alpha, h))v_l - \alpha v_h - (1 - \alpha)v_l + P' > 0$ and have trade with the consumer in period 2. Similarly, for any $P > (\phi(\alpha, h) - \alpha)\triangle v$ the firm who did not have trade in the first period can cut price such that $\alpha v_h + (1 - \alpha)v_l) - P' > \phi(\alpha, h)v_h + (1 - \phi(\alpha, h))v_l - P$. Thus the firm who did not have trade in the first period, can only post $P' = 0$.

When the observed quality of the output in the first period is low, the rival firm's reputation drops to zero. The expected utility from purchasing the good from the rival firm is v_l. The expected utility from purchasing the good from the firm who did not trade in the first period is $\alpha v_h + (1-\alpha)v_l$. Therefore in equilibrium, the only price the firm can charge is $\alpha v_h + (1-\alpha)v_l - v_l$.

Now consider the firm who traded with the consumer in the first period. Following similar arguments, if he produces a low quality output his reputation falls to zero. The expected utility from buying from his rival is higher and the only price he can charge in equilibrium is zero. On the other hand if he produces a good quality output in the first period, his reputation moves to $\phi(\alpha, h)$. The expected utility the consumer gets by purchasing from him is $\phi(\alpha, h)v_h + (1-\phi(\alpha, h))v_l$. The expected utility from the product of his rival is given by $\alpha v_h - (1-\alpha)v_l$. Therefore the price the firm can charge in equilibrium is $(\phi(\alpha, h) - \alpha)\triangle v$.

Given the belief update rule and price posting behavior we can now find the equilibrium effort level of the firm chosen by the consumer in the first period. If the opportunistic firm does not put effort in the first period, his reputation moves to zero and the only price he can charge in the second period is zero. However if he puts effort, he can charge the price P in the second period and the consumer in the second period has trade with him for sure. Thus the opportunistic firm is indifferent between putting effort and not putting effort in the first period if $P = c$. The indifference condition can be rewritten as

$$\left(\frac{\alpha}{\alpha + (1-\alpha)x^D} - \alpha\right)\triangle v = c \tag{7}$$

where x^D is the probability that the opportunistic firm exerts effort. Thus,

$$x^D = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{(1-\alpha)\triangle v - c}{c + \alpha\triangle v}\right) < 1 \tag{8}$$

Equilibrium Conditions: In equilibrium, the following conditions hold.

The firm who has trade in the first period charges P and his rival charges P' in the second period. The consumer in the second period purchases from the firm whose product yields higher net utility. In case of a tie the consumer purchases from the firm whose product yields higher expected utility.

In the first period, the effort choice of the opportunistic firm is given by,

$$x^D = \begin{cases} 0 & if\ (1-\alpha)\triangle v - c \leq 0 \\ \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{(1-\alpha)\triangle v - c}{c+\alpha\triangle v}\right) & if\ (1-\alpha)\triangle v - c > 0 \end{cases} \tag{9}$$

Proposition 2: *x^D is increasing in α and $\triangle v$ while decreasing in c.*

Proof:

x^D can be re written as

$$x^D = \frac{\triangle v - \frac{c}{1-\alpha}}{\frac{c}{\alpha} + \triangle v}$$

Partially differentiating x^D with respect to α, $\triangle v$ and c we get,

$$\frac{\partial x^D}{\partial \alpha} = \frac{\left(\triangle v + \frac{c}{\alpha}\right)\frac{c}{(1-\alpha)^2} + \left(\triangle c - \frac{c}{1-\alpha}\right)\frac{c}{\alpha^2}}{\left(\frac{c}{\alpha} + \triangle v\right)^2} > 0$$

$$\frac{\partial x^D}{\partial \triangle v} = \left(\frac{\alpha}{1-\alpha}\right)\frac{c}{(c+\alpha\triangle v)^2} > 0$$

$$\frac{\partial x^D}{\partial c} = \left(\frac{\alpha}{1-\alpha}\right)\frac{-\triangle v}{(c+\alpha\triangle v)^2} < 0 \qquad \blacksquare$$

Qualitatively the duopolist's choice of effort moves in the same direction as the monopolist's. Choice of effort is increasing in reputation below a threshold reputation $\alpha = \frac{\triangle v - c}{\triangle v}$. Above this threshold reputation the duopolist does not expend effort. As $\alpha \to 1$, $\phi(\alpha) \to 1$ and $(\phi(\alpha, h) - \alpha) \to 0$. Reputation revision becomes smaller and smaller as α approaches 1. Because of the presence of a rival, the only price the duopolist can charge at this range of reputation equals the difference between the expected quality of its own product and the expected quality of its rival's product. Thus gains from reputation shrink as the market becomes almost convinced about the firm's type and hence the duopolist does not have incentives to put effort for this range of reputation.

Reputation incentives are increasing in $\triangle v$ that is the increment in quality resulting from high effort as opposed to low effort. For a better understanding of the result, let us fix v_l and vary v_h. Note that in equation (6), for any given level of reputation $\alpha > 0$ such that $(1-\alpha)\triangle v - c > 0$, the price the duopolist can post in period 2, is increasing in v_h. On the other hand the gains from not putting effort is fixed at 0. Thus as the difference between the quality of the output increases reputation incentives become stronger. Notice that, given α and c the duopolist does not expend effort for a low range where $\triangle v < c/(1-\alpha)$. Above this range, as $\triangle v$ increases the duopolist's gains from building reputation increases as well.

The third result of Proposition 2 is straight forward. A higher cost of effort is associated with a lower choice of effort. Therefore, as c increases the threshold beyond which the monopolist puts effort with probability 1 increases too. The range of reputation where the monopolist exerts effort with positive probability is now characterized by lower level of effort.

3 Comparing monopoly and duopoly

In the previous section we analyzed the model for two cases namely, monopoly and duopoly. In this section we compare the results obtained in the previous section and see how reputation building behavior is affected by competition.

Define, $D = x^M - x^D$. Notice that $x^M > 0$ for all $\triangle v > 0$. Also, $x^M = 1$ for higher values of $\triangle v$. However, $x^D < 1$ for all parameter values and $x^D = 0$ for low range of $\triangle v$.

Theorem 1: *The equilibrium effort level is always strictly higher in monopoly, that is,* $x^M > x^D$.

Proof:

When $\triangle v \leq c/(1-\alpha)$, $x^D = 0$ and

$$x^M = \begin{cases} \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\triangle v - c}{c}\right) & if\ \alpha\triangle v < c \\ 1 & if\ \alpha\triangle v \geq c \end{cases} \tag{10}$$

Thus, $x^M > x^D$.

Now suppose, $c/(1-\alpha) < \triangle v < c/\alpha$.

$$x^M - x^D = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\triangle v - c}{c}\right) - \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{(1-\alpha)\triangle v - c}{c + \alpha\triangle v}\right)$$

$$= \frac{\alpha^2 \triangle v^2}{(1-\alpha)c(c+\alpha\triangle v)} > 0$$

When $\alpha\triangle v \geq c$, $x^M = 1$ and $x^D < 1$. Hence the proof. ■

The primary intuition for the above result is that price competition reduces future profit hence the value of putting effort. Thus gains from reputation are best reaped under monopoly. The presence of a rival who can cut price does not allow a firm to extract all the rent from reputation. This makes reputation building less rewarding under duopoly as compared to monopoly. To see this more formally, let $\triangle\pi^M$ and $\triangle\pi^D$ be the difference in profits generated by putting effort under monopoly and duopoly respectively. Now

$$\triangle\pi^M = \left(\frac{\alpha}{\alpha + (1-\alpha)x}\right)\triangle v$$

and

$$\triangle\pi^D = \left(\frac{\alpha}{\alpha + (1-\alpha)x} - \alpha\right)\triangle v$$

Thus $\triangle\pi^M > \triangle\pi^D$ for a given choice of effort and reputation. Thus the monopolist's benefits are higher than that of the duopolist from the same choice of effort. Hence the monopolist has higher incentives to put effort in equilibrium than the duopolist's choice of equilibrium effort level.

The following observations summarizes how the effort choice of a monopolist differs from that of a duopolist. Notice that under monopoly, the firm always puts effort with strictly positive probability and for some range the monopolist puts effort for sure. Under duopoly, for some range, the firm does not put effort at all and there are no range of parameter values for which the firm puts effort for sure. Also for the range $c/(1-\alpha) < \triangle v < c/\alpha$, $D = x^M - x^D$ is increasing in both $\triangle v$ and α.

4 Consumer arrives with probability 1/2

Since the model described in section 2 assumes that there is only one consumer in each period, the results we obtained in the previous section is also affected by the reduction in market size. In the monopoly model the market size for the firm was given by one consumer and the consumer-seller ratio was 1. In the duopoly model the ratio reduces to 1/2. In this section we analyze a model that isolates the effect of competition from the market size effect. This section assumes that the monopolist meets a consumer with probability 1/2 in the second period. This assumption ensures that the consumer seller ratio is the same regardless of the market structure.

Monopoly:

In the monopoly model there is one long lived firm and a new consumer in each period. In period 1, the consumer decides whether to buy from the monopolist and the monopolist decides on the quality of the product if the consumer is interested in trade. In period 2, a new consumer arrives with probability 1/2 who observes the quality of the good produced in the last period and updates her belief about the firm's type. The monopolist posts a price. The consumer then decides whether to buy from the monopolist.

Strategies: A stationary strategy for the firm is a pair (P, x), i.e. , a price and a choice of effort. Formally, $x : [0, 1] \to [0, 1]$, where $x(\alpha)$ is the probability that the opportunistic firm exerts effort in period 1 and $P : [0, 1] \to \mathbb{R}$ gives the fee the firm posts in period 2.

A consumer's strategy is whether to buy from a firm given her belief about the firm's type in the second period and the price the firm posts in the second period.

Belief update: Belief update rule is same as the belief update rule described in the section that assumes that the monopolist meets as consumer for sure. $\phi(\alpha, q)$ is the belief update function where, $\phi(\alpha, q)$ gives the probability that the firm is good given the prior α and the quality of output $q \in \{h, l\}$.

Clearly, $\phi(\alpha, l) = 0$. This is because the good firm produces a high quality output at no cost and the good firm is assumed to be non-strategic. Therefore, whenever the consumer observes a low quality output she concludes that the producer of the good is the opportunistic type. When $q = h$, the belief update function obtained using Bays rule is as follows

$$\phi(\alpha, H) = \frac{\alpha}{\alpha + (1 - \alpha)x}$$

where, α is the prior probability that the firm is good and x is the probability that the opportunistic firm exerts effort.

Analysis: Let us first consider the price posting behavior of the firm in period 2. In equilibrium, the firm posts a price that makes the consumer indifferent between purchasing the good and not purchasing it. The firm can do better if the price is such that the consumer strictly prefers trade to autarky. Hence the price must equal the expected utility the consumer obtains from the transaction and

$$P = \phi(\alpha, q)v_h + (1 - \phi(\alpha, q))v_l$$

Given the the belief update rule and the price posting behavior described above, the firm's gains from putting effort and producing a high quality good is given by $\frac{1}{2}\left[\frac{\alpha}{\alpha+(1-\alpha)x}v_h + \left(1 - \frac{\alpha}{\alpha+(1-\alpha)x}\right)v_l\right] - c$. Instead if the the firm does not put effort in period 1 and produces a low quality output, the consumer becomes sure that the firm is the opportunistic type. In this event the only price the firm can post in period 2 is v_l. Now since the firm meets a consumer in the second period with probability 1/2 the gains from effort is now lower than the previous case. The firm will be indifferent between exerting effort and not exerting effort if the following condition holds

$$\frac{1}{2}\frac{\alpha}{\alpha+(1-\alpha)x}(v_h-v_l)=c \tag{11}$$

Solving for x from the above equation we get,

$$x=\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\triangle v-2c}{2c}\right) \tag{12}$$

Equilibrium Conditions: In equilibrium the monopolist posts a price in period 2 which depends on the prior probability that the firm is good, the equilibrium choice of effort by the monopolist in period 1 and the observed quality of the output in period 1. Thus the equilibrium pricing rule is given by

$$P^M=\begin{cases} v_l & if\ q=l \\ \frac{\alpha}{\alpha+(1-\alpha)x^M}v_h+\left(1-\frac{\alpha}{\alpha+(1-\alpha)x^M}\right)v_l & if\ q=h \end{cases} \tag{13}$$

In equilibrium, the choice of effort of the opportunistic firm in period 1 is given by

$$x^M=\begin{cases} 0 & if\ \triangle v\leq 2c \\ \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\triangle v-2c}{2c}\right) & if\ \triangle v\in(2c,2c/\alpha) \\ 1 & if\ \triangle v>2c/\alpha \end{cases} \tag{14}$$

The equilibrium condition specified above implies that the monopolist does not exert effort for low range of $\triangle v$, puts effort with strictly positive probability in the middle range and puts effort with probability 1 for high range of $\triangle v$. Also x^M is increasing in $\triangle v$ for the range $\triangle v\in(2c,2c/\alpha)$.

Duopoly:

In the duopoly model there are two long lived firms and a new consumer in each period. Consumers trade only with one firm in each period. In period 1, the consumer chooses a firm and the chosen firm decides on the quality of the product. In period 2, a new consumer arrives who observes the quality of the good produced in the last period and updates her belief about the type of the firm who was hired in period 1. The firms post prices. the consumer then decides from whom to buy.

Strategies: A stationary strategy for the firm is a pair (P, x), i.e. , a price and a choice of effort. Formally, $x : [0, 1] \to [0, 1]$, where $x(\alpha)$ is the probability that the opportunistic firm exerts effort in period 1 and $P : [0, 1] \to \mathbb{R}$ gives the price the firm posts in period 2.

A consumer's strategy specifies from which firm to buy, given her beliefs about the the firms' types in the second period and the prices they post in the second period.

Consumer's belief and behavior: The belief update rule is same as described in the monopoly model. $\phi(\alpha, q)$ is the belief update function where, $\phi(\alpha, q)$ gives the probability that the firm is good given the prior α and the quality of output $q \in \{h, l\}$.

Clearly, $\phi(\alpha, l) = 0$. When $q = h$, the belief update function obtained using Bays rule is as follows

$$\phi(\alpha, H) = \frac{\alpha}{\alpha + (1 - \alpha)x}$$

where, α is the prior probability that the firm is good and x is the probability that the opportunistic firm exerts effort.

Given the belief update rule, the consumer in period 2 purchases only from a firm that gives her higher net expected utility. In case of a tie, we assume that the consumer purchases from the firm whose product yields higher expected utility.

The analysis and equilibrium condition is same as described in the earlier section. In the first period, the effort choice of the opportunistic firm is given by,

$$x^D = \begin{cases} 0 & if\ (1-\alpha)\triangle v - c \leq 0 \\ \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{(1-\alpha)\triangle v - c}{c+\alpha\triangle v}\right) & if\ (1-\alpha)\triangle v - c > 0 \end{cases} \tag{15}$$

Comparing monopoly and duopoly

First let us compare the monopoly results under the new set up with the monopoly results under the earlier set up. Notice that, in the new set up there exists a range of $\triangle v$ where the monopolist also does not put effort. In the earlier set up the monopolist would always put effort with strictly positive probability in equilibrium. However the monopolist under the new set up puts effort with positive probability for a high values of $\triangle v$ as he did in the earlier case and there exists a threshold beyond which he puts effort with probability 1. The monopolist puts effort with probability probability 1 for values of $\triangle v$ higher than c/α and $2c/\alpha$ under the old and new set up respectively. Thus the threshold beyond which the monopolist puts effort for sure is bigger when the consumer arrives with probability 1/2. Also in the range for which the monopolist puts effort with strictly positive probability, the choice of effort is strictly higher under the old set up. Therefore it's evident that market size does play an important role in determining the choice of effort and larger market shares lead to higher reputation incentives. Consequently a firm with a larger market share exerts more effort than a firm with a smaller market share. Because market size has such a considerable impact on the choice of effort in experience goods markets, it's worth isolating the impact of competition from the market size effect. Notice that the effort choice of the duopolist is same as earlier. The duopolist does not expend effort for a low range of $\triangle v$ and puts effort with strictly positive probability beyond a threshold. However the duopolist never puts effort with probability 1.

Proposition 3: *If $\alpha \geq 1/2$, the duopolist's choice of effort is no greater than the monopolist's choice of effort.*

If $\alpha < 1/2$ there exists a threshold $\overline{\triangle v}$ such that for $\triangle v < \overline{\triangle v}$ the monopolist's choice of effort is higher than the duopolist's choice of effort and, for $\triangle v > \overline{\triangle v}$ the opposite holds.

Proof:

From equation (14) and (15) we know that $x^M = 0$ for $\triangle v \leq 2c$ and $x^D = 0$ for $\triangle v \leq c/(1-\alpha)$.

Therefore, when $\alpha = 1/2$, the monopolist and the duopolist does not put effort for the exact same range.

When $\alpha > 1/2$ the range for which the monopolist does not put effort is smaller than the range for which the duopolist does not put effort.

Similarly, with $\alpha < 1/2$ the range for which the monopolist does not put effort is larger than the range for which the duopolist does not put effort.

Now define, $D = x^M - x^D$ and consider $\alpha \geq 1/2$.

For $\triangle v \in (c/(1-\alpha), 2c/\alpha]$, we know that both x^M and x^D are monotonically increasing in $\triangle v$. At $\triangle v = 2c/\alpha$, $x^M = 1$ and $x^D < 1$. Therefore for $\triangle v \in (c/(1-\alpha), 2c/\alpha]$, $D = x^M - x^D > 0$.]

For $\triangle v > 2c/\alpha$, $x^M = 1$ and $x^D < 1$. Hence, $D = x^M - x^D > 0$.

Now consider $\alpha < 1/2$.

$c/(1-\alpha) < 2c$.

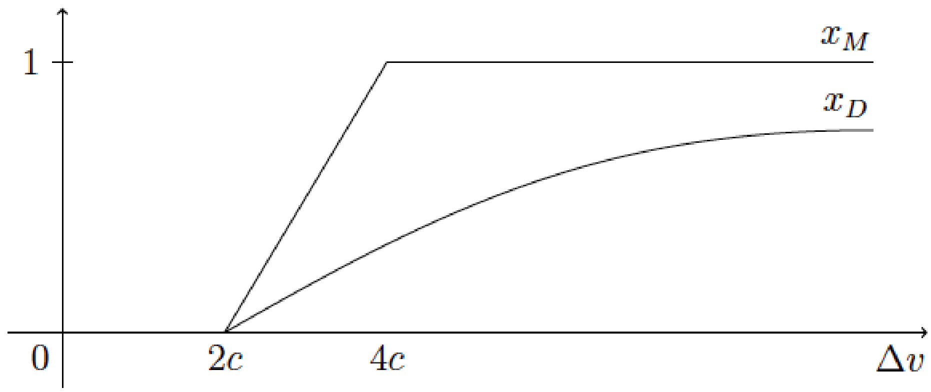
Thus, for $\triangle v \in (c/(1-\alpha), 2c]$, $x^M = 0$ and $x^D > 0$. Hence, $D = x^M - x^D < 0$.

Now at $\triangle v = c/\alpha$, $x^M = x^D = \frac{1}{(1-\alpha)} \frac{c-2c\alpha}{2c}$.

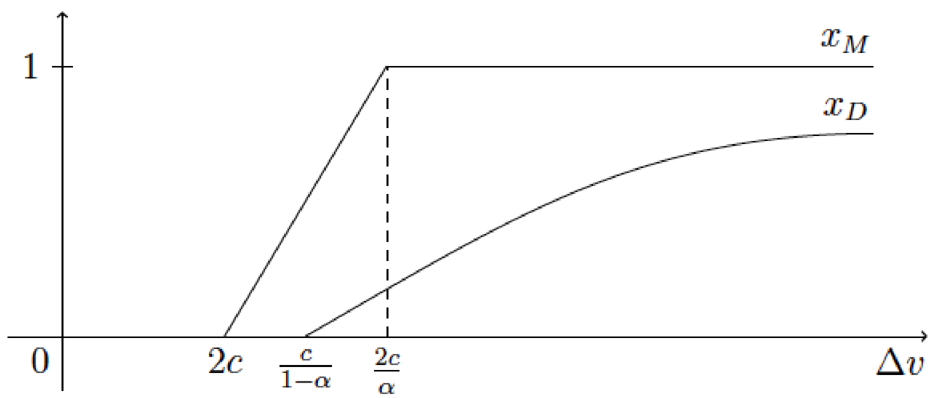
Since, x^M and x^D are monotonically increasing in $\triangle v$ for $\triangle v \in (2c, c/\alpha)$, $D = x^M - x^D < 0$.

Following similar logic as earlier, for $\triangle v > c/\alpha$, $D = x^M - x^D > 0$. ■

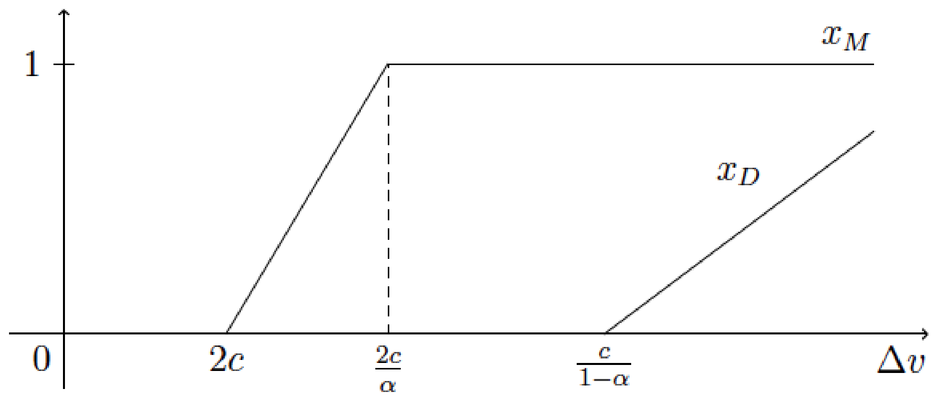
The following figures illustrate how the monopolist's choice of effort differs from that of the duopolist.



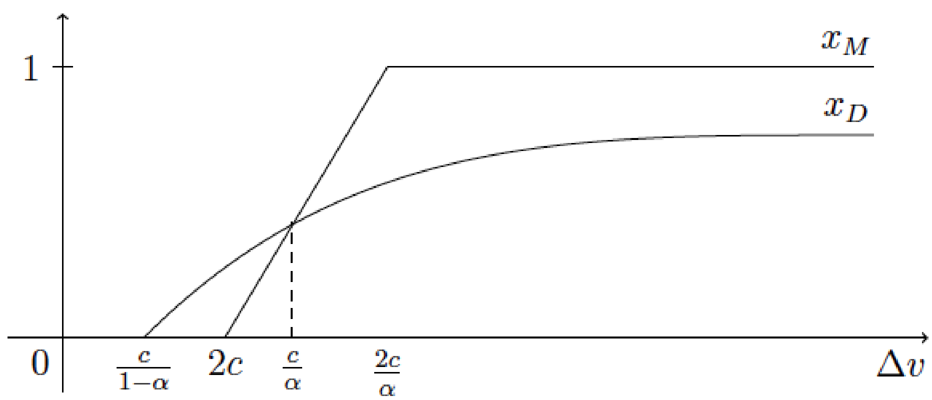
$Figure 1 : \alpha = 1/2$



$Figure 2 : \alpha > 1/2$



$Figure 3 : \alpha > 1/2$



$Figure 4 : \alpha < 1/2$

For a better understanding of the result let us consider the indifference conditions of the monopolist and the duopolist respectively.

$$\frac{1}{2}\left[\phi(\alpha)v_h + (1-\phi(\alpha)v_l - v_l\right] = c \tag{16}$$

$$\phi(\alpha)v_h + (1-\phi(\alpha))v_l - (\alpha v_h + (1-\alpha)v_l) - 0 = c \tag{17}$$

$\phi(\alpha)v_h+(1-\phi(\alpha)v_l$ in equation (16) depicts the returns of the monopolist from expending effort and v_l captures the return from not putting effort. In equation (17) the component that captures the duopolist's returns from putting effort is $\phi(\alpha)v_h+(1-\phi(\alpha)v_l-(\alpha v_h + (1-\alpha)v_l)$ and returns from not putting effort is 0. Notice that if v_l is high, punishment from not putting effort is low for the monopolist. Punishment for the duopolist on the other hand is always set to the maximum punishment, that is the duopolist loses his market in the second period if he does not put effort. Now let us fix v_h and vary v_l to see how the choice of effort moves in equilibrium as $\triangle v$ changes.

As $\triangle v$ increases, that is, as v_l decreases the monopolist's punishment from not putting effort decreases which in turn reduces his incentives to put effort. For the duopolist, a reduction in v_l reduces his rewards from putting effort, his punishment from not putting effort stays put on the other hand. Thus, a decrease in v_l leads to lower incentives to put effort for the duopolist as well. Now when α is high enough, the returns from putting effort is small for the duopolist. This is because, $\phi(\alpha)v_h+(1-\phi(\alpha))v_l-(\alpha v_h+(1-\alpha)v_l)$ is small for high values of α. The returns for the monopolist on the other hand is still positive. Thus the monopolist's incentive to put effort is higher than that of the duopolist for higher values of α. When α is sufficiently small and v_l decreases, that is, $\triangle v$ increases, the punishment for the monopolist is less severe. For the duopolist on the other hand punishment level remains the same. Thus the monopolist has weaker incentives to put effort as compared to the duopolist's incentives to put effort.

5 The planner's problem

In this section we discuss a planner's problem when the planner has "some" information about the hired firm's type. We are interested in a situation where the planner does not have "full" information about the firms' types and hence the first best can not be implemented. Since the cost of effort c is strictly less than the quality increment $\triangle v$, it is socially efficient for the opportunistic firm to expend effort whenever he has trade with the consumer. Now consider the following situation. The social planner has no control over the consumers' decision and the planner can only dictate the chosen firm's action in the first period. If the monopolist is opportunistic, the planner in the first period will dictate him to expend effort and produce a high quality output. In the second period the monopolist will choose to produce a low quality output.

Now under the duopoly set up suppose the planner only observes the chosen firm's type and he dictates the chosen firm's action. If the chosen firm is opportunistic and he produces high quality output by expending effort in the first period, the consumer fails to detect the opportunistic firm. Consequently the opportunistic firm is hired in the second period and produces low quality output. On the other hand if the opportunistic firm's type is revealed in the first period and its rival is good, then the output quality in the second period is high and the cost of producing the high quality output is zero. Thus in a duopoly set up it may not be socially efficient for an opportunistic firm to put effort in the first period. In order to understand the welfare consequences and to compare the choice of effort prescribed by the social planner with the choice of effort of a duopolist we consider the following model. In our model we assume minimum control of the planner.

Players and actions: There are two long lived firms, a social planner who lives only for the first period and a new consumer in each period. Consumers choose a firm and trades only with one firm in each period. In period 1, if the consumer chooses a firm , the planner observes the chosen firm's type. The consumer in the first period and the social planner share the same beliefs about the firm that has not been hired by the firm. The planner dictates the choice of effort by the chosen firm. In period 2, a new consumer arrives who observes the quality of the good produced in the last period.

Strategies: Strategy for the firm is a price $P : [0, 1] \to \mathbb{R}$ that gives the price the firm posts in period 2.

The Planner's strategy is a choice of action the planner chooses for the firm hired in the first period. The planners's strategy $x^P : [0,1] \times \{good,\ opportunistic\} \to [0,1]$ is a function of the chosen firm's type and the planner's belief about its rival.

A consumer's strategy specifies from which firm to buy, given her beliefs about the the firms' types in the second period and the prices they post in the second period.

Timeline: In period 1, if the consumer chooses a firm , the planner observes the chosen firm's type. The consumer in the first period and the social planner share the same beliefs about the firm that has not been hired by the firm. The planner dictates the choice of effort by the chosen firm.

In period 2, a new consumer arrives who observes the quality of the good produced in the last period and updates her belief about the type of the firm who was hired in period 1. The firms post prices. the consumer then decides from whom to buy.

Consumer's belief and behavior: The belief update rule is same as earlier. $\phi(\alpha, q)$ is the belief update function where, $\phi(\alpha, q)$ gives the probability that the firm is good given the prior α and the quality of output $q \in \{h, l\}$.

Clearly, $\phi(\alpha, l) = 0$. When $q = h$, the belief update function obtained using Bayes' rule is as follows

$$\phi(\alpha, H) = \frac{\alpha}{\alpha + (1-\alpha)x}$$

where, α is the prior probability that the firm is good and x is the probability that the opportunistic firm exerts effort. Notice that if the planner prescribes the chosen (opportunistic) firm to expend effort with probability 1, the consumer can not update her belief about the firm.

Given the belief update rule, the consumer in period 2 purchases only from a firm that gives her higher net expected utility. In case of a tie, we assume that the consumer purchases from the firm whose product yields higher expected utility. Also if firms are such that they yield same expected utility as well as same expected net utility, a firm is chosen at random with probability 1/2.

Analysis: Suppose the chosen firm in the first period is "opportunistic". If the planner dictates the firm not to put effort, the firm produces low quality output in the first period and his type is revealed. The consumer in the second period has trade with the rival firm and the expected quality of output in the second period is $\alpha v_h + (1-\alpha)v_l$. Thus the value the planner generates by dictating the firm in the first period to not put effort is $v_0 = v_l + \alpha v_h + (1-\alpha)v_l$.

If the planner dictates the opportunistic firm to put effort in the first period, the consumer in the second period fails to update her belief about the firm's type. Thus the opportunistic firm is hired in the second period with probability 1/2 and he produces output v_l in the second period. With probability 1/2 his rival is hired and the expected quality of output is $\alpha v_h + (1-\alpha)v_l$. Thus the value the planner generates by dictating the firm in the first period to put effort is $v_1 = v_h - c + \frac{1}{2}\left[v_l + \alpha v_h + (1-\alpha)v_l\right]$.

Now suppose the planner mixes with probability $\gamma = Pr(e=1)$, that is the planner dictates the opportunistic firm to put effort with probability γ. The social gain in the first period is $\gamma(v_h - c) + (1-\gamma)v_l$ and the social gain in the second period is v_l. Define $v_\gamma = \gamma(v_h - c) + (1-\gamma)v_l + v_l$.

Now v_γ is strictly increasing in γ. Define, $\overline{v}_\gamma = sup_{\gamma\in(0,1)} v_\gamma$.

Lemma 1: $v_1 > \overline{v}_\gamma$.

Proof:

$\overline{v}_\gamma = v_h - c + v_l$

$v_1 - \overline{v}_\gamma = v_h - c + \frac{1}{2}\left[v_l + \alpha v_h + (1-\alpha)v_l\right] - v_h + c - v_l = \frac{1}{2}\left[v_l + \alpha v_h + (1-\alpha)v_l\right] - v_l > 0.$

■

Therefore, whenever the planner dictates the opportunistic firm to put effort with positive probability he dictates the opportunistic firm to put effort with probability 1. Now our concern is under what circumstances the planner dictates the bad firm to put effort with probability 1. We need to compare v_0 and v_1 to obtain the effort level the planner prescribes.

Lemma 2: *When* $(1-\alpha)\triangle v > c$, $v_0 < \overline{v}_\gamma$.

Proof:

$v_0 - \overline{v}_\gamma = v_l + \alpha v_h + (1-\alpha)v_l - v_h + c - v_l = c - (1-\alpha)\triangle v.$

Thus if $(1-\alpha)\triangle v > c$, $v_0 < \overline{v}_\gamma$. ■

Proposition 4: *The duopolist's choice of effort either falls below or coincides with the choice of effort the planner prescribes.*

Proof:

Case 1: $(1-\alpha)\triangle v > c$

From Lemma 2, we know that $v_0 < \overline{v}_\gamma$ and from Lemma 1 we know that $\overline{v}_\gamma < v_1$.

Thus $v_0 < v_1$. The planner dictates the duopolist to put effort with probability 1.

However, from equation (15) we know that, $x^D = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{(1-\alpha)\triangle v - c}{c+\alpha\triangle v}\right) < 1$.

Case 2: $(1-\alpha)\triangle v \leq c$

From Lemma 2, we know that $v_0 \geq \overline{v}_\gamma$.

Now, $v_0 - v_1 = \left(1 - \frac{\alpha}{2}\right)v_l - \left(\frac{3}{2} - \alpha\right)v_h + c$ which is ambiguous.

When $v_0 > v_1$, the planner's solution coincides with the market outcome and the duopolist does not put effort.

When $v_0 < v_1$ the firm does not put effort under duopoly even if the planner would dictate him to put effort with probability 1. ■

6 Conclusion

In this paper, we provide an analytical framework and explicitly derive conditions for which competition aids reputation-building behavior and for which it hiders. This is a simple two-period model. Contrary to the prevalent idea that competition is conducive to reputation building behavior, our analysis suggests that under certain circumstances competition can hinder reputation building behavior. Consequently, competition has a negative impact on the over all quality of production. The intuition supporting this kind of result is as follows. Competition may reduce future expected payoff and hence can reduce incentives to put effort in the current period. This effect is more severe when the difference in values is large. Our result is robust to the market size effect embedded in imperfect competition models.

We also compare the market outcome under duopoly with that of a planner's solution. We show that the duopolist's choice of effort either falls below or coincides with the planner's choice of action. This analysis implies that there is inefficiency in the duopoly market whenever it is socially optimal for a duopolist to expend effort. On the other hand there is efficiency under duopoly when it is socially optimal for the duopolist to not expend effort.

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